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Overview of Wake-Vortex Hazards During Cruise

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A large amount of research effort has been devoted to the hazard posed by lift-generated vortex wakes of subsonic transport aircraft during approach and departure at airports. In contrast, only a small amount of effort has gone into the corresponding potential hazard at cruise altitudes. Because more frequent encounters with wakes may occur as the density of aircraft increases with time, improved knowledge is needed of the persistence and magnitude of the hazard posed by lift-generated wakes at cruise altitudes. The purpose here is to review and then to extend previous work to characterize properly the hazard posed by vortex wakes at cruise altitudes. Photographs are presented of the various fluid dynamic stages that vortex wakes usually go through as they age. Estimates made of the potential hazard that each stage poses indicate that the rolling-moment hazard during an axial encounter of a vortex wake can be as severe at cruise as for approach at airports; however, as in the airport-approach case, the hazard only persists for several minutes (about 20 n mile) behind the generating aircraft. However, the hazard posed by the downwash in the wake persists for up to tens of minutes (about 200 n mile) behind the generating aircraft. The downwash hazard is realized as severe vertical loads. A technique for avoiding vortex wakes at cruise altitude is described.

Nomenclature

A	= cross-section of wake, $\pi BD/4$
$\mathcal{R}$	= aspect ratio
a	= acceleration, ft/s^2
B	= breadth of visible wake, ft
b	= wingspan, ft
b'	= spanwise distance between vortex centers, ft
C_L	= lift coefficient, L/qS
C_l	= rolling-moment coefficient, M/qSb
c	= wing chord, ft
D	= depth of visible wake, ft
g	= acceleration of gravity, ft/s^2
L	= lift, lb
M	= rolling moment, ft · lb
n	= load factor, vertical acceleration/gravity
q	= dynamic pressure, $\rho U_\infty{}^2/2$, lb/ft^2
S	= wing planform area, ft^2
t	= time
U	= velocity of aircraft, ft/s
u, v, w	= velocity components in x, y, and z directions, ft/s
Wt	= weight of aircraft, lb
x	= distance in flight direction, ft
y	= distance in spanwise direction, ft
z	= distance in vertical direction, ft
α	= angle of attack, deg
Γ	= vortex strength, ft^2/s
ρ	= air density, slugs/ft^3

Subscripts

ail	= aileron
crz	= cruise
enc	= encountering aircraft
f	= following aircraft
g	= wake-generating aircraft
m	= maximum
v	= vortex
wk	= wake
o	= centerline
∞	= freesteam condition

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Introduction

THE hazard posed by and the persistence of lift-generated vortex wakes of aircraft in the vicinity of airports, especially during landing and takeoff, has been largely treated as an in-trail wake-vortex penetration problem (Fig. 1).[1–6] While aircraft cruise from one destination to another, both in-trail and across-vortex encounters with lift-generated vortices are likely to occur (Fig. 2). Examination of a bibliography by Hallock[7] indicates that a large amount of research effort, resulting in several hundred publications, has gone into the problem at airports and the in-trail penetration problem. In contrast, only about 15 publications are devoted to the across-wake or vertical-gust problem, which is likely during cruise.[8–22] Wake-vortex research programs devoted to the approach problem at airports concentrate on the in-trail or along-vortex encounter (Fig. 1) because it is the one that has a significant impact on airport capacity and safety. A solution to the wake-vortex problem at airports is needed because most hub airports are now at or near maximum capacity during significant parts of the day. Conversely, at this time, a solution to the wake-hazard problem at cruise altitudes would have a small, if any, impact on capacity. However, wake encounters at cruise can cause severe vertical and/or rolling accelerations that may disrupt flight paths and cause injury to passengers and crew.

Although the in-trail hazard posed by vortex wakes at cruise is small, the slowly spreading downwash, due to the lift on the wing of the wake-generating aircraft, can pose a vertical-loads hazard when aircraft cross through a wake during cruise. An increased likelihood of an across-trail encounter (Fig. 2) becomes significant when the persistence of the downwash component of lift-generated wakes of aircraft is included in the considerations. For example, a somewhat typical illustration of how vortex wakes persist and spread with time is shown by the photograph presented in Fig. 3. The narrow condensation trail was formed shortly before the picture was taken. The trail that is about four times as wide is estimated to be several minutes old. The condensation trail that looks like a feather (which occupies a large portion of the right-hand side of Fig. 3 and extends well outside of the photograph) is after about 1 h. The centerline of the wake is the dense trail where the two sides of the feather-shaped

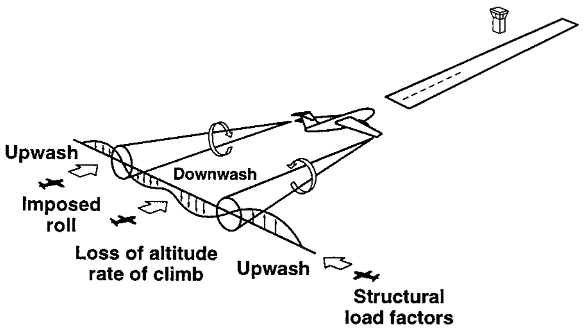


Fig. 1 Possible encounters by an aircraft with a lift-generated wake shed by a preceding aircraft while on approach to an airport.[6]

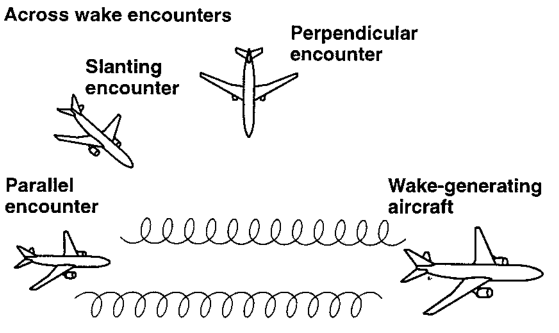


Fig. 2 Possible encounters by an aircraft while at cruise altitude with a lift-generated wake shed by a preceding aircraft.

Fig. 3 Condensation wakes shed by three aircraft at different times illustrating how vortex wakes age.

striations in the condensation wake join. Somewhere between the narrow wake and the one that has spread out into the shape of a feather, the downwash distribution becomes nonhazardous because wake activity has been spread laterally and vertically enough that the energy density of the wake is nonhazardous. Because of vertical and lateral spreading, the velocities in the wake are roughly inversely proportional to the depth or breadth of the wake. Because the time required for the downwash to spread sufficiently can take tens of minutes, a vertical-loads hazard persists in a long narrow region behind each aircraft as it cruises to its destination. When exhaust-condensation trails are not present, the whereabouts of hazardous wakes is not known, and other means must be used to locate them.

Even though flight paths during cruise are often in the same direction, wake-vortex encounters do not usually occur because their vortex wakes descend below the generating aircraft by 500 to 1500 ft as they age. However, across-wake encounters are still likely to occur. For example, it is suggested that some recent encounters[23,24] with clear-air turbulence might actually have been an across-trail penetration of a lift-generated wake. Such a suggestion is based on the fact that the aircraft mentioned in the two articles experienced large vertical accelerations for a very short period of time. No other atmospheric turbulence was experienced before or after the event. The presence of large vertical loads for a short duration indicates that the usual sources of clear-air turbulence were not present. This leaves the velocity field of a lift-generated wake as the one possible source for the vertical loads experienced by these two aircraft. When preliminary versions of this paper were presented to researchers more familiar with air traffic at cruise altitudes, comments were made by several individuals that they also had felt for some time that downwash in the wakes of aircraft was more than likely responsible for the short-duration high vertical gravity acceleration loads experienced in seemingly quiescent air.

If correct, incidents such as these will become more frequent as air traffic increases with time, and when the free-flight concept is implemented.[25,26] Under free-flight rules, aircraft will be free to choose their own path to a destination as the one most advantageous, which will probably result in more flight-path crossings than now occur. When vortex wakes are visible from the condensation of their exhaust gases, and it is daylight, the wakes can be identified early enough for pilots to avoid penetrating a wake. However, if atmospheric conditions are such that wake condensation does not occur, or the water/ice particles evaporate quickly, or darkness prevails, the presence of vortex wakes cannot be seen and intentionally avoided.

In the discussions that follow, the information about vortex wakes shed by aircraft is obtained by observing condensation wakes at cruise altitudes. A search of the literature[27–31] indicates that exhaust wakes become visible and persist when the local air temperature is below −40°F and the humidity is above 40%. Because temperatures at cruise altitude are generally sufficiently low, proper humidity is usually the controlling factor. It is also noted that vortex wakes are most persistent when cirrus clouds are also present. Most of the observations and photographs were obtained when persistence of wake condensation was good to borderline because other natural clouds were then not near the observation region. It was then reasoned that the condensation particles would be a reliable indicator of wake size, and possibly structure, because, just behind the wake-generating aircraft, the turbulence in the engine exhaust and the swirling flowfields of the vortices cause the two become thoroughly mixed. Any condensation that is spread outside of the downward moving vortex oval (which contains the vortex pair as it descends) is swept away by the flow past the outside of the oval. As a consequence, the condensation serves as a marker of air within the oval and associated with the vortex wake. In general, when condensation wakes disappear, it is because the ice crystals in them sublime, as part of the evaporation process. Because the condensation particles are ice crystals of very small size, they, like micrometer-sized water droplets, drift very slowly relative to the the air in which they are embedded. Because the energy contained in the condensation particles is small, it is expected that the dimensions of the wake as a function of time will be about the same whether or not condensation trails are visible.

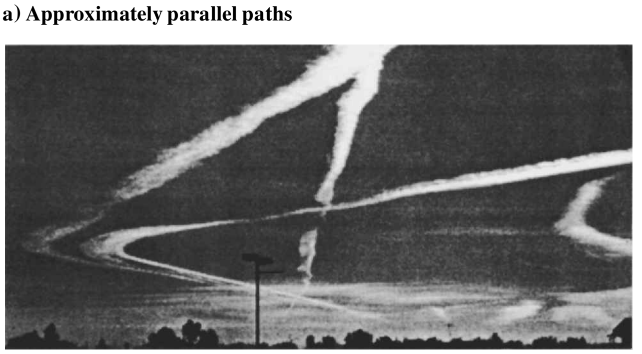
The purpose of the study reported here is to characterize properly the hazard posed by vortex wakes at cruise altitudes. To accomplish this goal, an overview is first given of information on in-trail and across-trail encounters of aircraft with vortex wakes that can be applied to cruise situations. Because existing theoretical methods used for computing vertical loads require an estimate of the structure of vortex wakes as they age, which is not available at this time, an alternate method based on conservation of momentum is introduced. A description with photographs is then given of the various stages that vortex wakes behind aircraft at cruise altitudes go through as they age. It is emphasized that the photographs in the text are not isolated incidents, but are representative of a large number of observations and photographs taken over several years in various parts of the country that demonstrated to the authors a consistent pattern in the formation and dispersion of condensation trails. Estimates based on photographs of condensation wakes are then made of the size and duration of the in- and across-trail hazards posed by each stage. The long duration of what appears to be an organized and coherent flow process in condensation trails was surprising. An outline is then provided for wake-avoidance procedures that could be used to prevent wake encounters while aircraft cruise to their destination.

Existing In-Trail Technology for Cruise

As evidenced by condensation trails in Fig. 4, aircraft often use about the same flight paths as they cruise between destinations. In-trail encounters with lift-generated wakes are rare, because the self-induced downward velocity of the wakes, and the winds across the flight path, move the wakes of preceding aircraft out of the way of following aircraft. The condensation trails shown in Figs. 4a and 4b have about the same direction and turning point in their flight paths and are offset largely by side winds. Even if cross winds are negligible so that the flight paths are aligned, both the time and distance between in-trail aircraft during cruise is usually much greater than used in the vicinity of airports. The added distance allows more time for vortex wakes to move and to decay.

Information on the in-trail hazard obtained for the airport problem is also applicable to vortices at cruise altitudes. For example, the change in the maximum swirl velocity in vortices with distance behind a variety of generating wings were measured by Ciffone and Orloff[32] in a water tow tank (Fig. 5). The data shown were the first

a) Approximately parallel paths



b) Aircraft executing a turn

Fig. 4 Condensation trails of aircraft at cruise altitude to illustrate approximately the same flight paths between destinations; offset between trails is at least partially due to side winds.

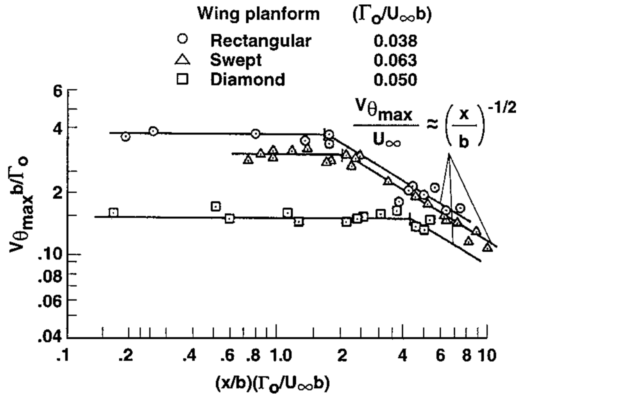


Fig. 5 Water tow-tank results for the maximum swirl velocity as a function of downstream distance behind wings of different planforms; α_g = 5 deg and U_∞ = 6.8 ft/s (2.07 m/s) (Ciffone and Orloff[32]).

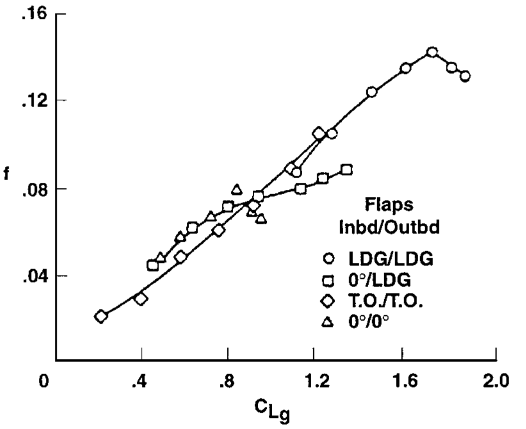


Fig. 6 Maximum rolling-moment coefficients on following wings in wake of B-747 for various flap settings as measured in 40 × 80 ft wind tunnel; x_f/b_g = 13 (Corsiglia et al.[34] and Corsiglia and Dunham[35]).

to indicate a decay process wherein no significant decay occurs for some distance after the wake was generated. Thereafter, the peak swirl velocity decays roughly as $1/t^{1/2}$. As an extension of this work, data were gathered by Iversen[33] for a wide variety of wings from data taken in flight, water tow tanks, and wind tunnels. By use of special functions to correlate the data, the change in the maximum swirl velocity with downstream distance behind the wing, or time, was shown to be quite similar for a wide variety of wings and test conditions. The results are significant in that the plateau region does not end until the parameter $(x/b)(\Gamma_0/U_\infty b_g)$ exceeds about 2 (Fig. 5). Because values of $\Gamma_0/U_\infty b$ are about 0.1 for landing configurations, the data indicate that about 20 wingspans of travel are required before vortex decay begins in the airport environment. Because decay or diffusion is based on time, it is expected that the plateau region ends, and decay begins, at about [20(550/150) = 70 spans] or at 3 miles of travel. Of course, decay of the wake to a harmless level takes much longer, as will be shown later.

Ground-based wind-tunnel and water tow-tank measurements were made with a variety of combinations of configurations at in-trail distances up to 1 mile.[34–38] These measurements[34,35] indicate that the rolling moment for three configurations of the B-747 used in the test all lie roughly along a common line (Fig. 6). Even though the lift coefficients on the wake-generating aircraft are less, that is, about $C_{Lg} = 1$ during cruise, than the $C_{Lg} \approx 1.5$ values used during landing, the wake-induced rolling moments are still greater than the aileron-induced rolling moments available on most subsonic transports. Another set of measurements,[37] which has been confirmed by calculations and by the observation of pilots, illustrates the vortex-imposed in-trail rolling-moment changes with the ratio of the span of the following wing to that of the wake-generating wing, b_f/b_g (Fig. 7). Note that the following aircraft is just able to cope with the vortex-induced torque when the span ratio, b_f/b_g, is one. Aircraft

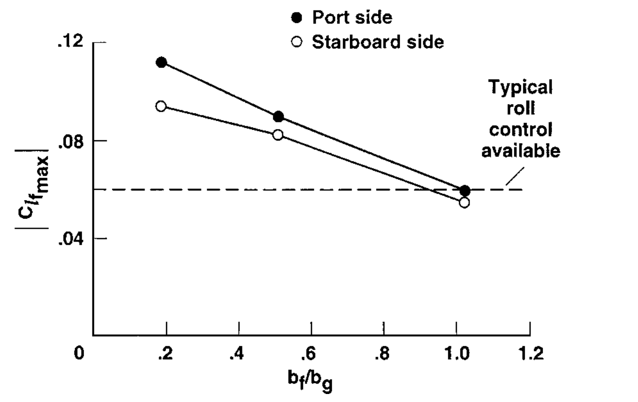


Fig. 7 Wake-induced maximum rolling-moment coefficients as a function of ratio of span of following wings to span of B-747 as measured in 80 × 120 ft wind tunnel[37]; B-747 landing flaps set at (30 deg, 30 deg), x_f/b_g = 13.

with span ratios less than one will, therefore, experience overpowering rolling moments should they encounter a wake shed by a larger aircraft. The general characteristics and magnitudes measured in ground-based facilities for the encounters have been confirmed in flight tests.[6]

Because the information presented in Figs. 5–7 is for a limited set of combinations of wing spans, approximate closed-form expression were derived for the general case.[39] Study of the structure of vortex wakes indicates that the most intense rolling moment occurs when the centerline of a small aircraft is aligned with the center of a vortex. Furthermore, the largest vortex-induced lift is induced on the encountering aircraft when its wingtip penetrates the center of the vortex. (The vortex-induced rolling moment is also severe for such an encounter.) Simple closed-form relationships for these conditions have been derived for the maximum lift- and rolling-moment coefficients as[39]

$$C_{Lf})_m = [\mathcal{R}_f](\Gamma_v/b_g U_\infty)(b_g/b_f) \tag{1}$$

$$C_{lf})_m = [\mathcal{R}_f/8](\Gamma_v/b_g U_\infty)(b_g/b_f) \tag{2}$$

These two relationships identify the significant parameters and their functional dependence for the determination of loads during an in-trail wake encounter. The magnitude of the two maximums in Eqs. (1) and (2) are noted to be the same except for a factor of eight.

The two formulas are quite accurate for small aspect ratios, that is, less than one, but quickly become less accurate as aspect ratio increases above one. It has been pointed out by Jones and Cohen[40] that the lift-curve slope for a wing of small aspect ratio is given by $dC_L/d\alpha = \pi\mathcal{R}_f/2$ and that, at larger aspect ratios, another relationship given by

$$\frac{dC_L}{d\alpha} = \frac{2\pi\mathcal{R}_f}{p\mathcal{R}_f + 2} \tag{3}$$

is much more accurate. This simple formula, which is convenient and surprisingly reliable, was derived by Jones (e.g., Ref. 40, page 95). It is, therefore, recommended that the quantity, $(2dC_L/d\alpha)/\pi$ be substituted for $\mathcal{R}_f$ inside each of the square brackets in Eqs. (1) and (2). In Eq. (3), the quantity $\mathcal{R}_f$ is the aspect ratio of the wing, p is the ratio of the semiperimeter of the wing to its span, and $dC_L/d\alpha$ is the lift-curve slope. Because the maximum lift on the wing occurs when the tip of the encountering wing is aligned with the impinging vortex, all of its span is involved with either upwash or downwash from the vortex. The various parameters in Eq. (1) should then be based on the entire span of the encountering wing. If, however, the impinging vortex is aligned with the center of the encountering wing for maximum induced rolling moment on the encountering wing, it is found that the parameters in Eq. (2) should be based on either the port or starboard half of the encountering wing, as if it were isolated from the other half.[36]

The freestream velocities used in the ground-based tests are close to those used for landing and takeoff of aircraft. Because once formed vortices decay as a function of time rather than distance traveled by the wake-generating aircraft, changes in the vortices should be scaled with time rather than distance. Note that the rolling moments induced on following wings by the vortices as measured in the tests did not change appreciably for distances of 1 mile in the wind tunnel, and for about $1\frac{1}{2}$ miles in a tow tank. Therefore, it is expected that not much change will occur behind the wake-generating aircraft in cruise for distances of at least

$$\text{plateau distance} \approx 1.5(550/150) = 5.5 \text{ n mile} \tag{4}$$

where 550 kn is taken as the cruise velocity and 150 kn as the approach velocity.

All observational data indicate that the distance is appreciably more than 5 miles, but does not indicate a limiting distance. Observations made of condensation trails, which will be discussed later, indicate that about 2–5 min are required for mutually induced instabilities to render a vortex wake incoherent and nonhazardous for in-trail encounters. This observation is supported by several examples. A severe rolling-moment encounter at cruise altitudes occurred during a NASA program devoted to the study of chemical constituents in jet exhaust gases by using one aircraft to probe the wakes of other aircraft. During one such measurement, the probe aircraft encountered an in-trail vortex segment that was still coherent enough to cause the probe aircraft to roll about 60 deg, even with full counterroll input by the ailerons. The encounter with the vortex occurred at 8 n mile behind the wake-generating aircraft, or at about 1 min of flight time. During another flight, roll excursions not quite as severe were also encountered at distances as large as 15 n mile. The persistence of coherent vortex wakes behind the generating aircraft, and the instabilities that lead to disintegration of the coherent structure of the vortices, is discussed in a section to follow.

Existing Across-Wake Technology

When an across-vortex encounter does occur within several miles behind the wake generator, the aircraft experiences severe vertical loads of short duration brought about by the up- and downwash flowfield of the vortices in lift-generated wakes. At several miles behind the wake generator, vortices are still compact with concentrated cores, and organized up- and downwash flowfields (Fig. 1). McGowan[8] was one of the first to recognize the potential hazard posed by vortex wakes of aircraft and then to proceed to estimate the various loads that vortex wakes impose on aircraft that encounter them. His paper also provides results of an extensive set of computations that illustrate the dynamics and vertical loads to be experienced by the encountering aircraft. His computations apply mainly to encounters with relatively newly generated vortex wakes. The method used to estimate the instantaneous loads is based on those described by Jones,[9] Pierce,[10] and Kordes and Houbolt,[11] and gust-response theories adapted to perpendicular encounters with vortex wakes. McGowan[8] assumes that the span loading on the encountering wing is elliptic, which is a good representation when the angle of encounter is perpendicular to the path of the wake-generating aircraft. At other angles of encounter, the time-dependent lift is not symmetrical about the wing centerline so that both vertical and rolling-moment accelerations are induced.

McGowan's[8] method calculates the instantaneous lift on the penetrating wing from the lift-curve slope for the wing and the instantaneous angle of attack induced on it as it crosses the wake. The up- and downwash is based on a model for the vortex wake represented by the superposition of a pair of Rankine vortices (Fig. 8). The instantaneous angle of attack is calculated as a sum of the steady state and the instantaneous angle of attack induced by the vortex pair on the wing of the penetrating aircraft. The time-dependent lift at each wake station is then adjusted at each across-trail location of the wing for the so-called lag-in-lift effect by use of the Kuessner function (e.g., see Jones[9]) (Fig. 9). A correction to the instantaneous lift is needed because time is required for the flowfield to establish and deestablish the circulation in the flowfield for the lift. One manifestation of lag-in-lift is that an across-trail vortex is shed at the trailing edge of the wing whenever the lift changes. The vortex shed at the trailing edge is opposite in sign to the lift increment experienced by

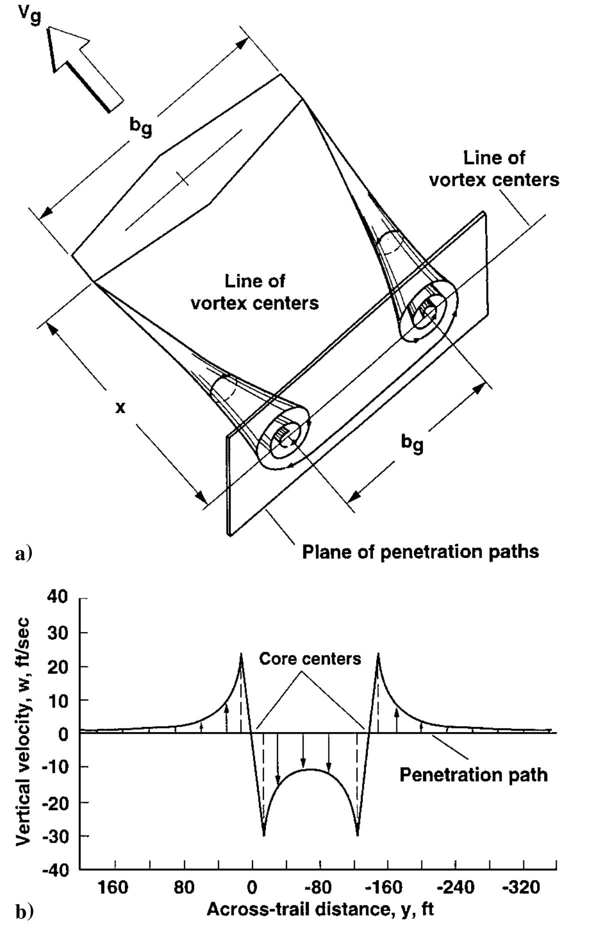


Fig. 8 Wake model used by McGowan[8] to calculate expected vertical loads on aircraft during 90-deg encounter with lift-generated wake of another aircraft: a) diagram of wake model and plane of penetration path and b) vertical velocity distribution assumed for wake structure (Rankine-vortex pair).

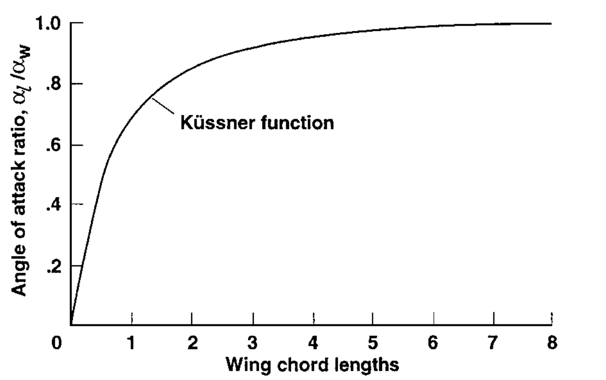


Fig. 9 Function used to determine effective angle of attack on wing as it penetrates a sharp-edged gust (i.e., lag-in-lift or Küssner effect) (McGowan[8] and Jones[9]).

the wing, thereby reducing the instantaneous lift through a reduction in the effective angle of attack. The Küssner function (Fig. 9) indicates that about 90% of full lift is achieved after about three chords of travel and that about 98% is achieved after six chords of travel into a sharp-edged gust. Because the angle of attack induced on the encountering wing by the flowfield of the vortex pair is constantly changing, a stepwise procedure is used along with the Küssner function to estimate the rapidly changing loads on the wing as it crosses a vortex wake.

McGowan's analysis[8] also assumes that the wing/flowfield is linear and irrotational except for the shed vorticity. Such an assumption is equivalent to assuming that the flowfield is a potential one wherein the free-stream is uniform. The other part of the equivalence assumption is that the wake-induced changes in angle of attack are accomplished by instantaneous changes in the wing angle of attack as it crosses the wake. Such an interpretation points out that the method of computation assumes that the vertical momentum in the airstream is unlimited, whereas the rotary flowfield of the vortices has only a finite contribution to the local vertical momentum.[41] It is known that such an assumption can lead to an overprediction of the vertical loads by up to about 30% when the span of the penetrating wing becomes equal to the span of the wake-generating wing.[41] Because the methods described in the following paragraphs make the same implied assumptions, their theoretical results are believed also to overpredict the vertical loads by about the same factors when the span of the penetrating aircraft is about the same as the wake width. However, the computed predictions are probably more accurate than knowledge of the distribution of the vertical velocities in the wake. Because the loads estimated here are not intended to be precise, any errors due to the various approximations are ignored.

By use of the method just described, McGowan[8] provides several examples of across-wake encounters to illustrate the magnitude of the hazard posed by various situations. The estimates are made for the loads and the dynamics experienced by three aircraft either as wake generators, or as wake penetrators, as each crosses at right angles through one of the three vortex wakes.[8] The three aircraft, A, B, and C, chosen for McGowan's across-vortex study weigh 180,000, 295,000, and 400,000 lb, respectively, and have wing spans equal to 174, 142, and 90 ft. Aircraft A represents propeller-driven subsonic transports, and B represents jet-driven subsonic transports. Aircraft C is a supersonic design. Figure 10 presents the results of McGowan's computations for the loads, $n = a_z/g$, where a_z is the vertical acceleration experienced by the center of gravity of the aircraft and the acceleration of gravity is used to normalized the loads. Note that more than 1 g acceleration in both upward and downward directions, that is, above that caused by the normal acceleration of gravity, is experienced in all cases even when a small aircraft crosses the wake of an identical aircraft. When aircraft A crosses the wake of the largest aircraft in the sample, even the ultimate design loads

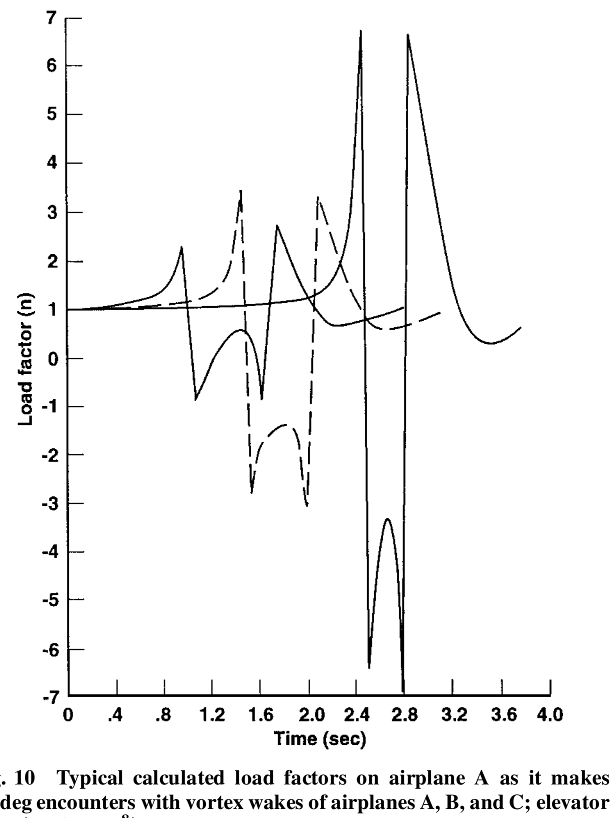


Fig. 10 Typical calculated load factors on airplane A as it makes 90-deg encounters with vortex wakes of airplanes A, B, and C; elevator fixed (McGowan[8]).

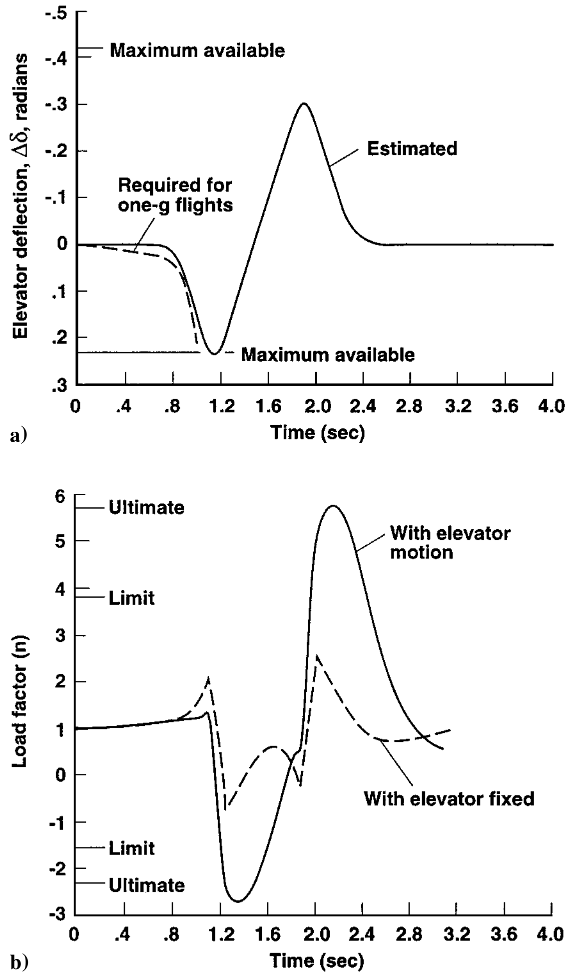


Fig. 11 Load factors on airplane A as it makes a 90-deg encounter with vortex wake of another airplane A (McGowan[8]): a) estimated elevator input as response by pilot and b) calculated load factors with and without pilot input into elevator angle.

for the wing are exceeded. Such an incident occurred near an airport when a single engine general aviation aircraft crossed the wake of a transport aircraft. The encounter caused both wings to fail. Other less devastating across-wake events have also occurred during the past 30 years. These incidents tend to confirm the estimates made by McGowan[8] and serve to indicate the seriousness of across-wake encounters in the near field of the vortex wake history that exists before vortex-wake instabilities have had time to reduce the intensity of the swirling velocities in the vortices.

McGowan[8] also analyzes the dynamics when the elevator of the penetrating aircraft is used by a pilot to alleviate the wake-induced loads. It was found (Figs. 11a and 11b) that the loads change so rapidly that control inputs tend to aggravate the predicted accelerations by substantial amounts rather than to alleviate them. Even under ideal circumstances, the elevator control available is not able to provide the needed response (Fig. 11a). The time needed to rotate the aircraft enough to compensate for the vortex-induced angles of attack on the wings is usually longer than the encounter time with the wake. McGowan also computed paths of the encountering aircraft during and after wake penetration by assuming both that the controls were fixed and that the controls were deflected to alleviate the loads. It was recommended that the encounter be executed with controls fixed because the time-dependent loads oscillate so rapidly that pilot and aircraft response are usually of such a phase that their input aggravates rather than alleviates the loads and the dynamics of the aircraft.

Some work by the Royal Aircraft Establishment in England reported by Rose and Dee[12] and by Bisgood et al.[13] confirms the general characteristics of an encounter as predicted by McGowan's computations.[8] References 12 and 13 provide flight measurements on across-wake penetrations by a Lincoln aircraft (66,000 lb) in the wakes of a Comet 3 and a Vulcan 1 aircraft (100,000 lb). The measured data indicate the kinds of trajectories that penetrating aircraft can experience. Because the weight ratios between the wake-generating and penetrating aircraft are not as large as those considered by McGowan, the vertical loads are around 0.3 g both upward and downward from steady-state flight, which is not as extreme as predicted by McGowan. Because the measurements were all made within several minutes behind the wake-generating aircraft, data were not obtained on the persistence of the hazard posed by vertical loads.

Condit and Tracy[14] decribe results from an extensive test program conducted by The Boeing Company on the potential hazard of aircraft in their cruise and landing configurations. Aircraft types include an F-86, B-737, B-707-320C, B-747, CV-990, and C-5. The paper is of value not only for the encounter data but also for the data on the changes in vortex wakes with distance behind wake generators. Comments made by pilots on the response of their aircraft as they encountered the wake are included.

Methods for calculating the loads induced on aircraft making across-wake penetrations are also described by Houbolt[15] and by Jones and Rao.[16] Houbolt's[15] theoretical estimates indicate that vertical loads above steady flight in excess of 2 g are likely if the encountering aircraft is much smaller than the generator and about 0.4 g when they are both about the same size. The examples considered by Jones and Rao[16] assume that the penetrating aircraft are about two vortex spacings above the plane of the vortex centers. As a consequence, the encounters are not expected to be as severe. Jones and Rao's method is such that encounters from any approach angle can be analyzed. In both the method of Houbolt[15] and of Jones and Rao,[16] the loading on the encountering wing was determined by use of a Fourier series representation of the span loading similar to that described in chapters X and XI of Glauert.[42] In this way, an improved representation of the span loading on the penetrating wing is achieved as it moves across the wake in a slantwise direction (Fig. 2). The method of Jones and Rao[16] was also applied by Ramirez et al.[17] to the encounter of a DeHavilland Beaver with a vortex wake shed by a B-747 at an encounter angle of 5 deg. Results are presented for the magnitude of the vertical- and rolling-moment loads along with the longitudinal stability of the penetrating aircraft.

Nelson[18,19] presents an analysis that permits computation of the response dynamics of aircraft as they encounter vortex wakes at a wide variety of encounter angles from axial to perpendicular encounters. He states that trial computations indicate that lifting-line/strip theory provides about the same results as more complex theories, indicating that the spanwise loading is probably well represented by both models. The simpler load-predicting method made it possible to carry out simulations of encounters on a six-degree-of-freedom model for three aircraft sizes, that is, business aircraft (15,000 lb), DC-9 (70,000 lb), and Convair 990 (153,000 lb). The computations present data for near axial encounters at downstream distances up to 10 miles and for perpendicular penetrations. In one example, computations for an across-wake penetration indicate vertical loads for the business jet in the wake of a DC-9 that alternately exceed 1 g in less than a second. As with McGowan,[8] Nelson[18,19] also concluded that stick-fixed loads are usually less than those where load alleviation is attempted. McWilliams,[20] Britton,[21] and Pinsker[22] also present information on wake encounters while aircraft cruise between destinations.

In all of the foregoing theoretical models, the structure of the vortex wake that was used in the examples was one like those found within a mile or two behind the wake-generating aircraft, that is before the mutually induced instability or some other wake-modifying mechanism made substantial changes in the structure of the vortices. At the time of those studies, the structure of lift-generated vortex wakes as they age was not well defined. At this time, some measurements are available for the relatively near field behind aircraft. However, after the wake has undergone the mutually induced, or Crow[43] instability, which includes vortex linking and the subsequent mixing in the wake, no information is available on the velocity field of the wake and only observations of condensation trails are available. Because the region from about 3 n miles and more behind the wake-generating aircraft is of interest for the purposes of this investigation, neither the wake structure nor the up- and downwash distributions in vortex wakes are known well enough for load estimates to be made. As a consequence, a more approximate and global approach is described in the next section.

Load Estimates Based on Momentum Transfer for Across-Trail Penetrations

The flowfield model to be used in the derivation of expressions for the vertical loads experienced by an aircraft as it makes a crossing encounter with a vortex wake is shown in Fig. 12. The loads are evaluated by developing relationships for the downward momentum contained in the lift-generated wake of the wake-generating aircraft and for the momentum transferred to the encountering aircraft, thereby giving it a downward velocity. The derivation begins by observing that, to maintain straight and level flight, the wake-generating aircraft imparts downward motion to a given volume of air per unit time. The rate at which downward momentum is generated is related to the weight of the aircraft in steady flight by

$$Wt_g = \rho U_g A_{\text{gwk}} \Delta w_{\text{gwk}} \tag{5}$$

where ρ is the local density of the air and U_g is the velocity of the wake-generating aircraft. The cross-sectional area of the wake at the time of generation is A_{gwk}, where the subscript gwk is a combination of the generator and wake subscripts to denote the wake immediately behind the wake-generating aircraft. The quantity Δw_{gwk} is the increment in downward velocity imparted to the air in the wake by the lift on the wing.

Now turn to relationships for the loads to be expected when an aircraft encounters the downward momentum in a wake in a direction across the wake at an angle θ_{enc}, as shown in Fig. 12. It is assumed that the downward momentum in the lift-generated wake is transferred to the penetrating aircraft so that

$$\Delta M_{\text{enc}} = \Delta M_{\text{wk}} \tag{6}$$

It is next assumed that the downward momentum in the wake is spread uniformly throughout the wake, so that the momentum transferred can be evaluated by use of geometric parameters involved in the encounter. Because the cross section of the wake enlarges as it ages, the portion of the wake intercepted by the encountering aircraft will decrease as the wake depth increases. It is assumed that the penetrating aircraft passes through the entire wake and that it intercepts all of the momentum along its path, that is, an increase in the breadth of the wake does not change the total momentum transferred. The momentum intercepted then imparts to the penetrating aircraft a downward velocity increment given by

$$\Delta M_{\text{enc}} = Wt_{\text{enc}} \Delta w_{\text{enc}} \tag{7a}$$

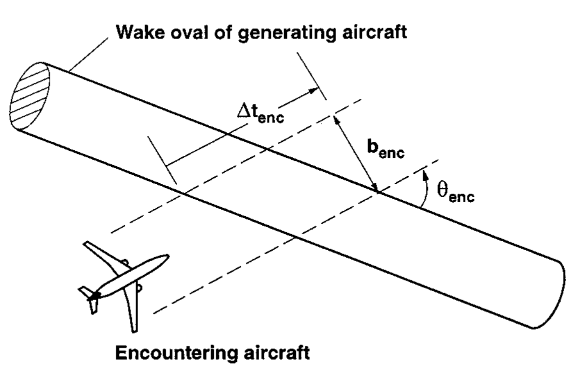


Fig. 12 Across-trail encounter of aircraft with lift-generated wake to illustrate momentum-transfer concept used to estimate vertical loads.

The increment of downward momentum intercepted by the penetrator aircraft is given by:

$$\Delta M_{\text{wk}} = g\rho \frac{b_{\text{enc}}}{\sin\theta_{\text{enc}}} \frac{b_{\text{enc}} A_{\text{wk}}}{D_{\text{wk}}} \Delta w_{\text{wk}} \tag{7b}$$

If the downward momentum in the wake is conserved as the wake ages and spreads,

$$A_{\text{wk}} \Delta w_{\text{wk}} = A_{\text{gwk}} \Delta w_{\text{gwk}} \tag{8}$$

where the subscript wk denotes evaluation of the parameter at the time of the encounter, then the wake cross section will probably be larger than when originally generated, that is, denoted by the subscript gwk. Equation (8) assumes that the density of the air stays constant with time.

Combination of Eqs. (5–8) leads to the total increment in downward velocity given to the penetrating aircraft by the downwash in the wake as

$$\Delta w_{\text{enc}} = \frac{g}{U_g} \frac{Wt_g}{Wt_{\text{enc}}} \frac{b_{\text{enc}}^2}{D_{\text{wk}} \sin\theta_{\text{enc}}} \tag{9}$$

To develop an equation for the loads on the penetrating aircraft during the encounter, the time interval Δt_{enc} over which the downward momentum is assumed to be transferred is determined as

$$\Delta t_{\text{enc}} = \frac{B_{\text{wk}} + b_{\text{enc}} \cos\theta_{\text{enc}}}{U_{\text{enc}} \sin\theta_{\text{enc}}} \tag{10}$$

The vertical acceleration of the aircraft, as it encounters a wake, is then given by $a_{\text{enc}} = \Delta w_{\text{enc}} / \Delta t_{\text{enc}}$. Combination of Eqs. (9) and (10) yields

$$\frac{a_{\text{enc}}}{g} = k \frac{U_{\text{enc}}}{U_g} \left(\frac{b_g^2}{D_{\text{wk}}(B_{\text{wk}} + b_{\text{enc}} \cos\theta_{\text{enc}})} \right) \left(\frac{Wt_g}{b_g^2} \frac{b_{\text{enc}}^2}{Wt_{\text{enc}}} \right) \tag{11}$$

where a quantity k is used as an adjustment factor for any item not correctly represented in the global analysis that led to Eq. (11). In the present study, comparison with other estimates indicates that $k = 1$ appears to be applicable.

Equation (11) identifies the four ratios that govern the magnitude of the vertical-loads hazard posed by a wake. The first quantity listed is the ratio of the velocities of the encountering aircraft to that of the wake-generating aircraft, which is probably always close to one for subsonic transports. The second quantity represents the portion of the wake intercepted as the size of the wake increases during its aging process (Fig. 13).

The third and fourth ratios have been combined into a single set of brackets because they generally offset one another for subsonic transport aircraft, causing their combination to be about one. Therefore, even though they each can vary considerably (e.g., the ratio can go from 0.1 to 10 if the two aircraft involved are a business jet

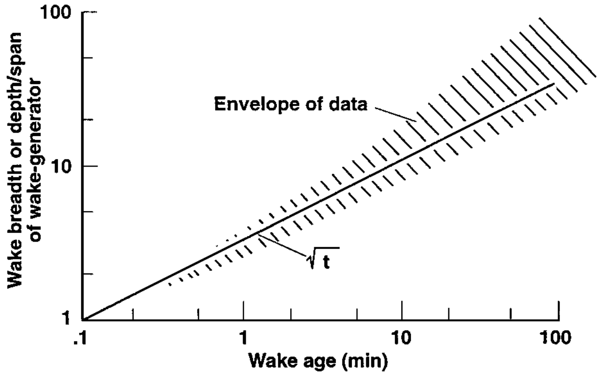


Fig. 13 Approximate breadth or depth dimension of condensation wakes at cruise altitudes as a function of time.

Table 1 Stages of vortex decay

Stage number	Aerodynamic process	Duration of stage	Rolling moment	$\frac{a_{\text{enc}}/g}{(Wt_g/Wt_{\text{enc}})(b^2_{\text{enc}}/b^2_g)}$
1	Rollup of vortex sheet shed by generating wing	0–10 s	$>C_{l\,\text{ail}}$	1
2	Wake divides and instabilities occur	0–2 min	$>C_{l\,\text{ail}}$	1
3	Vortex loops fade and wake parts rejoin	2–5 min	$<C_{l\,\text{ail}}$	0.5
4	Wake reorganizes itself into coherent vortex pair	5–15 min	$\ll C_{l\,\text{ail}}$	0.2
5	Condensation striations develop	15 min–1 h	$\ll C_{l\,\text{ail}}$	0.01
6	Condensation trail loses identity	After 1 h	$\ll C_{l\,\text{ail}}$	<0.01

and a large transport), the variations largely offset one another for subsonic transport aircraft so that

$$\left[\left(Wt_g/b_g^2\right)\left(b^2_{\text{enc}}/Wt_{\text{enc}}\right)\right] \approx 1 \tag{12}$$

This result comes about because subsonic transports have about the same wing aspect ratio and fly at the about same lift coefficient and cruise velocity. The weight ratio and the span ratio squared, therefore, combine for a constant factor of roughly one. The two ratios will be kept in combination because of their special relationship, but will not be set equal to one because different types of aircraft are in the air fleet, for example, both subsonic and supersonic transports. As a consequence, the combination of ratios presented in Eq. (12) is not always one, for example the three aircraft used in McGowan's examples.[8]

To find out whether Eq. (11) provides realistic results or not, estimates by use of Eq. (11) are compared with calculations made by use of the more complete analysis carried out by McGowan.[8] Assume that the adjustment factor k is one, and that the encounter is at an angle perpendicular to the wake, that is, $\theta_{\text{enc}} = 90$ deg. Also, because McGowan assumed a downwash velocity distribution that applies to newly generated wakes permits the assumption that the second factor in Eq. (11) may be taken as one. Consider the encounter of the smallest aircraft with wakes generated by all three of his aircraft samples. When the smallest aircraft A crosses the wake of an identical aircraft, the load factor predicted by Eq. (11) is one. McGowan's computations indicate maximum loads of around 1–2 g (Fig. 10). Similarly, when the small aircraft crosses the wake of aircraft B, Eq. (11) predicts maximum loads of 2.5. McGowan's results indicate maximum values of vertical loads in the upward direction as slightly over 2 and in the downward direction as around 3, which is in good agreement considering the assumptions made in the two theories. In the case of the small aircraft penetrating the wake of the heaviest aircraft sample used by McGowan, the vertical acceleration predicted by Eq. (11) is 8, whereas McGowan's method (Fig. 11) predicts about 5.5 g upward and between 7 and 8 g downward. These comparisons indicate that, if $k = 1$ is assumed, reasonable estimates are provided by the momentum-transfer method when vortex wakes are compact and have not yet experienced diffusion through-wake instabilities.

Rate of Wake Spreading

Data obtained by observation of the condensation trails associated with vortex wakes are now used to determine the rate at which vortex wakes spread or grow in cross section as they age. The graph of measured wake sizes presented in Fig. 13 is not very precise for various reasons. The wakes were usually viewed at an angle and from substantial distances. Also, the span of the wake-generating aircraft was assumed to be the same as the initial size of the wake, which is only approximately true. Also, no information was available as to the size or type of the aircraft that generated the wake being observed. It was also found that the time required for a wake to change from one stage to the next varied between wakes observed and between locations on the same wake. As a consequence, the wake-size changes in a lift-generated wake indicated in Fig. 13 should be considered as approximate.

The origin of the graph in Fig. 13 is placed at 0.1 min and one span. For comparison purposes, a line is drawn from the origin at 0.1 min and one span with a slope corresponding to a wake spreading rate of $t^{1/2}$. The $t^{1/2}$ function was chosen because vorticity, circulation, and the core radius all spread at that rate for a laminar vortex according to Lamb's solution[44] for an isolated line or two-dimensional vortex, which is given by

$$v_\theta = (\Gamma_0/2\pi r)[1 - \exp(-r^2/4\nu t)] \tag{13}$$

where v_θ is the swirl velocity in the vortex, Γ_0 is the entire circulation content of the line vortex, and ν is the kinematic viscosity of the fluid.

Stages of Wake Decay During Cruise

To gain a better understanding of the way that lift-generated wakes disperse with time, the stages that vortex wakes go through as they age are now discussed and illustrated by means of photographs of visible exhaust condensations of aircraft flying at cruise altitudes. Before going into detail on each stage of decay, Table 1 presents an overview of the fluid-dynamic stages. The observations are identified with a descriptive title for each stage, an estimate of the duration of each stage of wake decay, and of the maximum loads to be experienced during an encounter. Time is used rather than distance behind the wake-generating aircraft because vortex strength and time (not distance) determine wake structure. (For a relationship between time and distance, an aircraft flying at 550 kn moves at about 9 n mile/min, or 900 ft/s.)

In Table 1, the rolling-moment hazard is indicated as to whether or not the aileron capability, $C_{l\,\text{ail}}$, onboard the encountering aircraft is greater than the torque induced on the encountering aircraft by the vortices in the wake; that is, does the aircraft have enough roll-control power with its ailerons to cope with the vortex or not. The column that lists an estimate of vertical loads to be expected during a 90-deg or perpendicular crossing of the wake are made with Eq. (11) on the basis of wake sizes estimated for condensation wakes as they age. In the loads estimates, the flight velocities of the two aircraft are taken as equal. Because, as indicated in Table 1, the relationship indicated by Eq. (12) is not always one, the quantity in Eq. (12) must be considered in most encounters where the two types of aircraft involved in the encounter are not certain. In fact, improved accuracy will probably be obtained if the ratio given in Eq. (12) is factored into the estimates, when the various parameters are known.

Note that, beyond some time, the lift-generated wakes become so large and diffuse that they do not pose any kind of hazard. Before that time, it is difficult to place a definite time at which the wake becomes safe to penetrate at any angle. It is reasoned that safety cannot be assured until the fifth stage is well under way because even 0.2 g of vertical load on a standing or walking passenger, not expecting it, can cause a fall. The numbers listed in the two right-hand columns need to be explored by flight experiments to determine if their magnitudes and times of occurrence are conservative.

Stage 1: Rollup of Vortex Sheet (0–10 Seconds)

The appearance of condensation wakes during the first several minutes after being shed by subsonic transport aircraft are shown in Figs. 14. The portion of the wake that includes the region where the vortex sheet shed by the wing rolls up into a vortex pair is located in a small segment at the left end of the wake shown in Fig. 14a. An estimate of the time required for rollup of the vortex sheet shed by the wing can be obtained from data available on wakes shed by

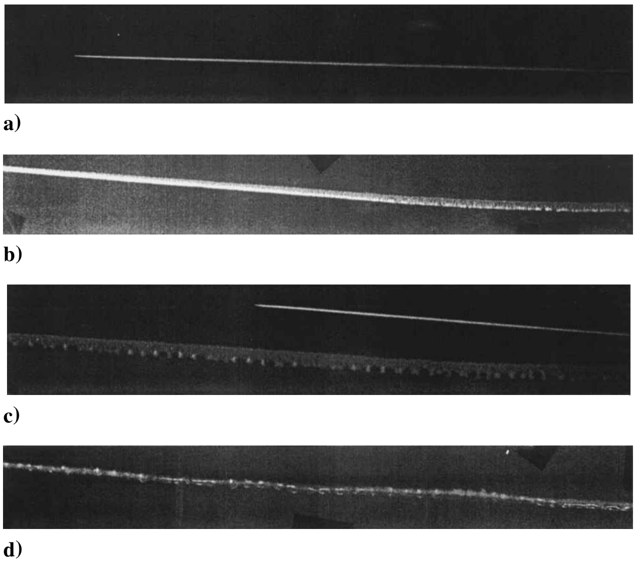


Fig. 14 Photographs of condensation wakes during early stages of change: a) condensation wake of aircraft as it is being generated; b) 1 min after photograph a, side view of wake division, instabilities begin; c) vortex loops from Crow instability[43] grow in size, about 2.5 min after photograph a, new vortex trail above old wake indicates rapid growth of vertical depth of wake; and d) view of another wake from below illustrating how vortex loops are not centered on wake; wake about 1 min old.

aircraft getting ready to land at an airport. Under high-lift conditions at an airport ($C_{Lg} \approx 1.5$), the wake can be considered as rolled up for most purposes within about 5 spanlengths (less than 0.1 min) behind the generating wing. If it is assumed that the lift on an aircraft in cruise is about $C_{Lg} = 1.0$, the circulation in each vortex is estimated by the equation for lift written as

$$L = \rho U_\infty \Gamma b'_g \tag{14}$$

The ratio of roll-up times between airport approach and cruise for a given aircraft is then

$$\Gamma_{\text{crz}} / \Gamma_{\text{ldg}} = (\rho_{\text{ldg}} / \rho_{\text{crz}})(U_{\infty\ \text{ldg}} / U_{\infty\ \text{crz}}) \tag{15}$$

which yields

$$\Gamma_{\text{crz}} / \Gamma_{\text{ldg}} \approx (1.0/0.25)(150/550) \approx 1.0 \tag{16}$$

Because the time required for wake rollup is proportional to the strength of the circulation in each vortex, the time required for rollup at cruise is the same as that at landing. However, because aircraft move faster at cruise, the distance required is 5 spans $\times$(550/150) or approximately 3.7 times as far as during approach, which is equal to about 18 spans of travel distance at cruise. Note that 18 spans of travel is about 0.1 of the wake shown in Fig. 14a. If a wingspan of 200 ft is chosen, because it represents the larger aircraft in the fleet, the distance is about 0.6 n mile, which is far less than distances that aircraft trail one another. Therefore, the length of the condensation wake during which rollup is, for all practical purposes, completed is less than 1 in., as shown in Fig. 14a. Therefore, for aircraft in the current transport fleet, the vortex wake can be considered as fully rolled up before any expected encounter could occur.

If, however, an in-trail encounter should occur before the wake is rolled up or decomposes appreciably, wind-tunnel measurements presented in Figs. 6 and 7 indicate that overpowering wake-induced rolling moments will occur if the wake-generating aircraft is larger than about one-half of the size of the penetrating aircraft.[37,38] Similarly, if an across-trail penetration is made, both McGowan's results[8] and Eq. (11) indicate that short-duration, vertical loads on the order of 1 g or more will occur, depending on the weight and span ratios of the two aircraft. It is concluded, therefore, that the rollup region of the wake and a considerable distance beyond it are to be avoided.

Stages 2: Wake Divides and Instabilities Occur (0–2 Minutes)

Features that might be associated with the development of the long-wavelength instability within the wake are not yet noticeable in the visible wake during rollup of the vortex sheet nor for some distance thereafter (Figs. 14a and 14b). The first significant change in the shape of the condensation wake is that a port and starboard vortex filament appear below the bulk of the condensation wake (Fig. 14b). [The vertical or cirumferential striations around the wake (Figs. 14a and 14b) that sometimes occur will be discussed subsequently.] The appearance of a pair of vortex filaments below the bulk of the condensation cloud makes it appear that the wake is dividing into two parts with one below the other. Because the velocity fields of the vortices extend well beyond their visible exteriors, both parts of the wake continue to communicate with each other.

It is usually during the time of wake division (Figs. 14b and 14c) that the long wavelength version of the mutually induced instability for a vortex pair (or Crow instability[43]) begins to develop. Shortly after the vortex filaments emerge beneath the exhaust condensate, antisymmetrical, transverse waves form on the originally nearly straight filaments, and then grow. As presented by Crow,[43] and shown in Fig. 15, the amplitude of the waves grow quickly until they approach each other closely at the troughs of the waves. The wavelength of the instability is about 6–8 wingspans. When the two vortices are in close proximity, they link across the space between them to form a sequence of irregularly shaped vortex loops. Crow called the foregoing process a mutually induced instability because the velocity field of the vortex pair induce and amplify wave-type disturbances that exist as turbulence in the atmosphere. A vortex pair and disturbances of finite amplitude are required because a vortex filament by itself does not have the instability.[43,45] Other self- and mutually induced vortex motions and instabilities also occur. The one studied and correctly explained by Crow[43] involves a vortex pair and is also the largest instability so far identified in vortex wakes.

Several other unidentified forms of instability (some of which may be mutually induced) quite often manifest themselves shortly after the vortex wake has been formed. The phenomenon appears on the exterior of the condensation trails as if fluid was being squirted out

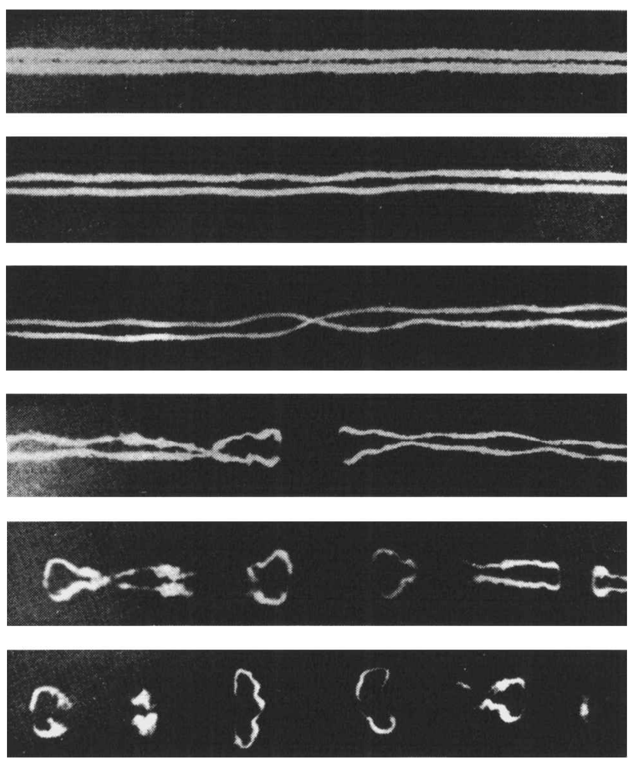


Fig. 15 Photographs taken from below wake at 15-s intervals of condensation wake of B-47 in cruise configuration to illustrate mutually induced instability of a vortex pair (Crow[43]).

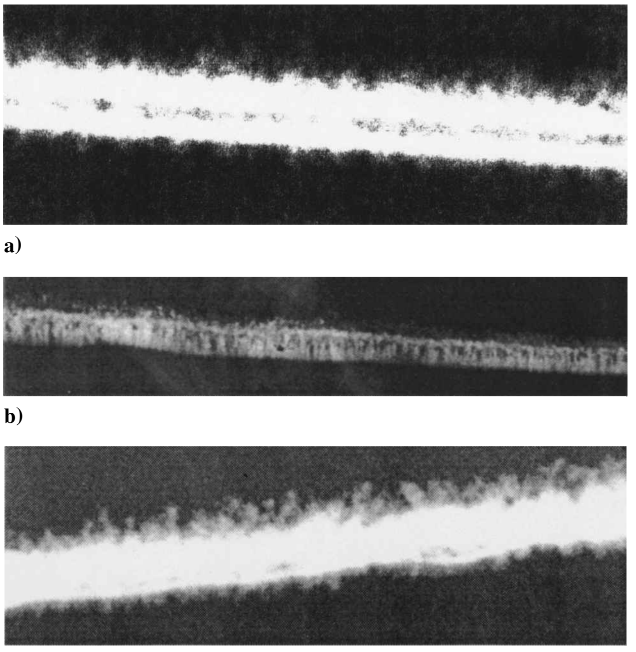


Fig. 16 Photographs of condensation wakes that appear to exhibit other forms of wake instabilities and large-scale mixing: a) condensation wake of aircraft with port and starboard parts due to outboard-mounted engines that illustrate spike or brushlike proturberances from main condensate clouds, wake about 5 s old; b) side view of wake exhibiting circumferential striations without division; and c) vortex loops protruding from upper surface of vortex wake during early stages of wake division, wake about 0.5 min old.

of the condensation trail (Figs. 16). The extent of the protrusions is expected to be short lived because a vortex oval generally encloses the vorticity in the vortex pair and moves downward under its own self-induced velocity field. Because such a flowfield does not have a shear-layer interface on the oval, the origin of the brushlike features suggest an instability or turbulence on the outside of the condensation cloud that is not expected. The striations in Fig. 16a extend about one span length beyond the concentrated parts of the wake in needlelike protuberances that extend outward from the surface of the condensate in such large numbers that, from a distance, the wake surface has a coarse brushlike appearance. Although the feature has been identified by previous observers,[12–14] the three-dimensional aerodynamic process that causes it has not. It is observed that the bristles that protrude from the surface of the condensate are numerous and well defined and that they occur early in the wake history. Their small needlelike features indicate that it is not likely that they are formed from or are associated with the large eddies that occur from the linking and dynamic decay of the vortex loops generated by the mutually induced instability. Engine-generated eddies are a possible source for the needlelike features, but the mechanism is not clear. It seems that the most likely explanation might be related to an energy-difference instability, that is, high-energy fluid inside the oval that makes local penetrations into the ambient fluid across the interface between the vortex oval and the ambient fluid.

The striations around the outside of the wake condensation cloud is often but not always present (Figs. 14 and 16). Based on some direct Navier–Stokes computations by Laporte et al.,[46] it is suggested that the circumferential striations on the exterior of the condensation wake is a result of the short wavelength mutually induced instability studied by Crouch[47] and Leweke and Williamson.[48] Comparison of the circumferential striations in Fig. 16b with the computational presentations[46] indicates that the striations could be a result of the short-wave instability that has grown to a more mature stage.

Both Figs. 16b and 16c indicate clearly the appearance of a number of fairly large vortex-eddy type of protrusions on top of the concentrated part of the wake. Visually, these protrusions appear to be vortex loops or eddies of condensate. The eddies may result

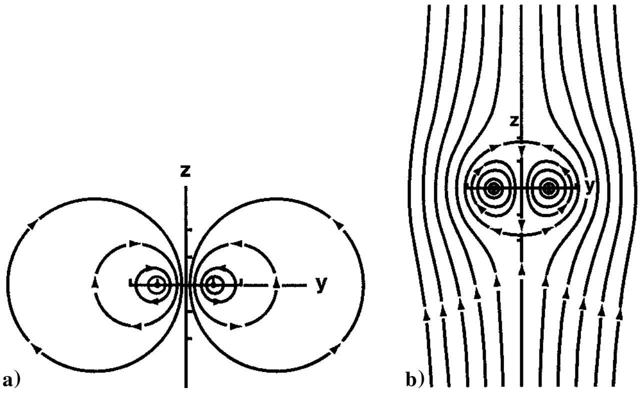


Fig. 17 Streamlines for the steady-state flowfields of two equal and opposite point vortices: a) vortices forcibly held in place; and b) vortices moving under self-induced velocity-field.

from a mutually induced instability that occurs in the vorticity that remains within the main exhaust cloud after the vortex filaments have moved downward to execute the Crow instability.[43] Such an explanation seems plausible because departure of energetic vortex filaments down out of the condensate oval leaves a residue of vorticity or circulation in the main body of condensate. The residue of circulation must also undergo a mixing process that produces considerable disturbance to the vorticity or circulation left behind (estimated to be about 30–40% of the total). In other words, the departure of the vortex filaments downward out of the primary vortex oval of condensate to carry out the large-scale mutually induced instability causes moderately scaled eddies to form within and around the oval, as well as below the oval, where the vortex loops from the mutually induced instability form and decompose. These smaller eddies might be the ones that move mostly in upward and sideways directions as well as downward. The eddies, jets, etc., produce mixing that quickly spreads the vorticity over increasingly larger cross sections of the wake. Not only does the mixing action spread out the circulation, but because the vortex oval has a self-induced downward motion, any vorticity or circulation transferred to the outside of the oval is swept away by the velocity field around the oval (Fig. 17). This process is probably the reason why the global model of wake decay derived by Greene[49] has been so successful in predicting the decrease in intensity of a vortex pair as it ages. Greene's model assumes that the skin friction and flow-separation drag of a representative shape for the wake is proportional to the loss of circulation by the two vortices. Because the velocity across the surface of the vortex oval is continuous, skin friction is not expected. The mixing across the oval interface due to flowfield instabilities and to large-scale eddies transfers fluid and vorticity that can be substantial and is probably responsible for wake vorticity being swept into the flowfield outside of the vortex oval contour.

Figures 16b and 16c appear to have a long-wave character of periodic downward condensation elements, which suggests that perhaps a weakened form of the mutually induced instability is beginning to take place, as illustrated with the lower wake shown in Fig. 18. Another nearby wake slightly above the lower wake was quite similar in appearance several minutes earlier and continues to spread quickly. The two wakes diffuse to form the configurations shown in a closeup photograph in Fig. 18. Note that the upper condensation trail is not a smooth or uniform condensation cloud but an assembly of clumps of condensate. The photograph suggests that the eddies in vortex wakes persist long after the mutually induced instabilities have gone to completion. These eddies no doubt contribute to the continued rapid spreading of the wake.

Now consider the in-trail hazard posed by a vortex wake during the beginning stage of decomposition (Figs. 16 and 18). Before the onset of the mutually induced instability, the rolling-moment hazard posed by the wake is just as severe as described in the preceding section. However, after the instability has gone to completion, the wake no longer appears to contain a coherent rolling-moment hazard. A significant in-trail hazard is not expected because spacings of

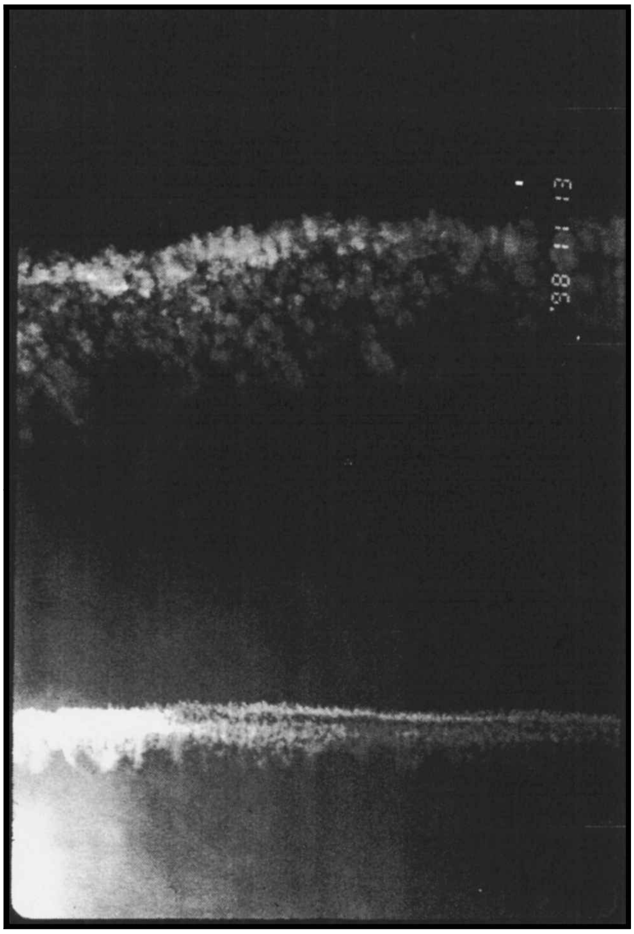


Fig. 18 Lower wake with circumferential striations and some evidence of mutually induced instability, about 1.5 min old, and upper wake that began to age in the same way and is now about 5 min old.

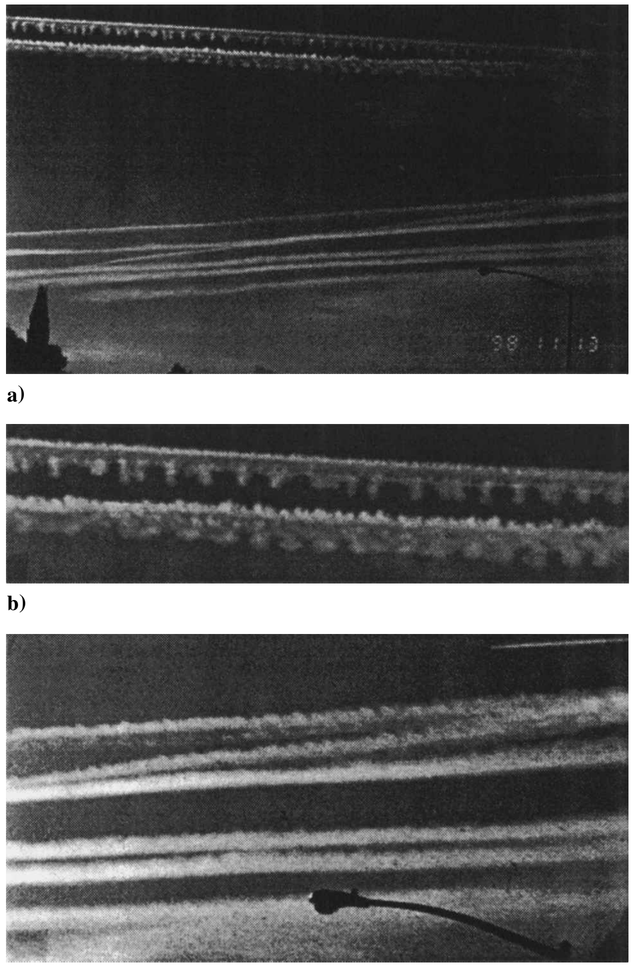
transport aircraft during cruise are usually more than about 3 min, which is just about always long enough for the decomposition of vortex pairs to be completed before a following aircraft arrives. Therefore, during this stage of wake decomposion, the in-trail hazard goes from a very severe level to an almost benign level. However, caution should be still be exercised when certain atmospheric conditions exist because the mutually induced instability may not occur, in which case a rapid decomposition of vortex wakes does not happen.[35,45,49] A severe rolling-moment hazard posed by the wake may then persist for longer than a few minutes behind the wake-generating aircraft. Observations of the exhaust condensate indicate that other mechanisms or instabilities (e.g., Figs. 16 and 18) then take over and cause the organized intense wake to decompose to one that is also less hazardous to in-trail penetrations. The alternative processes may, however, take longer to render the wake incoherent than the 3–5 min used by the mutually induced instability.

Several severe in-trail encounters occurred at cruise altitudes during a NASA flight program devoted to the study of chemical constituents in exhaust gases by using one aircraft to probe the wakes of other aircraft. During one such measurement, the probe aircraft encountered an in-trail vortex segment that was still coherent. The encounter caused the probe aircraft to roll about 60 deg despite full counterroll input by the ailerons. The encounter with the vortex occurred at 8 n mile behind the wake-generating aircraft, or about 1 min of flight time. During other flights, roll excursions that were not quite as severe were also encountered at distances as large as 15 n mile, or just under 2 min after wake formation.

During the instability stage of wake change with time, it is estimated that the wake usually grows in both depth and breadth by at least a factor of four in the first minute or so. On occasion, however, wakes such as the one shown in Fig. 16a do not appear to change appreciably in size for 1–3 min. After that, the mutually induced instability sets in, and the wake size changes rapidly.

Stage 3: Vortex Loops Fade and Wake Parts Rejoin (2–5 Minutes)

After the loops of vortex filaments have descended below the bulk of the condensation trail and have undergone the mutually induced instability, the intensity of their cores appears to fade so that the condensation wake takes on the appearance of limp and loose loops hanging down below the upper part of the wake (Fig. 19). The limp appearance suggests that the vortex filaments involved in the mutually induced instability are much less intense and have become benign. The upper and lower parts of the exhaust condensate then appear to rejoin to form a single trail (Figs. 19 and 20). Sometimes



a) b) c)

Fig. 19 Side views of vortex wakes at ages from new to over 0.5 h: a) overview of wakes; b) closeup view of two upper wakes aged about 1 and 2 min; and c) enlarged view of lower wake cluster in photograph a from new (top right) to over 1 h in age.

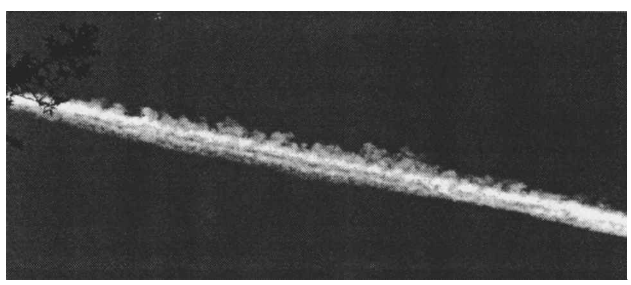


Fig. 20 Side view of wake about 1.5 min old illustrating large number of strong eddies protruding from top of wake; vortex filaments also depart from and rejoin bottom of wake.

the process of rejoining proceeds quickly, as it did in Fig. 20, and other times it appears to take a long time. At first, it was thought that buoyancy forces might be responsible for the process. It is now believed that the circulation in the two parts of the wake resembles, in end view, two vortex pairs that leap frog each other for about one-half cycle. Hence, the velocity field of the two pairs causes the lower wake to divide and pass around the sides of the upper part and to mix with it. This reorganization of the two parts is often not uniform or ideal but asymmetric because the linking and loop formation is not always perfectly symmetrical. Instead, as shown in Fig. 14d in a wake viewed directly from below, the vortex loops are not always centered on the rest of the wake, but meander port and starboard. As a result, the amount of vorticity on each side of the wake varies along the length of the trail because the links of vorticity have passed around and up one side or the other. As a consequence, the final vortex wake often appears to rotate and to change in width along its length.

As the vortex loops lose their intensity the limp hanging loops disappear from side views because they are convected around the sides of the wake, and the visible wake is reformed. Observations of condensation trails confirms that after 3–5 min (about 25–50 n mile) the intensely swirling parts of vortex wakes appear to have been rendered into an incoherent flowfield, and the vortex loops formed by the mutually induced instability lose their identity. At this stage of wake decomposition, wakes should not produce significant rolling moments on aircraft making an in-trail penetration.

Some direct evidence exists that indicates that a significant rolling-moment hazard does not exist after the 3–5 min of mixing just described has occurred. The evidence is provided by subsonic transports that occasionally fly both nearby and within decomposed wakes. Passengers who enjoy looking out of windows while traveling can easily determine when such a penetration occurs. It is first noted that their aircraft is flying parallel to, and off to the side of, a decomposed wake made visible by condensation of exhaust gases. A short while later, an announcement comes over the public address system stating that turbulence is soon to be expected and that everyone should take their seats and buckle up. The aircraft then moves over into the decomposed wake (5–10 min in age) and the predicted bumpy ride begins. The ride has a somewhat periodic character to it much like that experienced in a car going over a washboard gravel road common in the midwest. The periodicity probably comes from the period associated with the wavelength and linking of the vortices through the long-wavelength version of the mutually induced instability. After a while, the bumpiness ceases as the aircraft leaves the wake. In the limited amount of experience of the author on these occasions no significant rolling moment was experienced. Events such as this show that, once decomposed, the aircraft does not encounter a significant wake-induced rolling moment, but rather senses the large-scale turbulent character of the wake.

After the flight, discussions with the pilots reveal that the penetration was deliberate, and that the only significant effect on the penetrating aircraft (besides a moderate response to turbulence) is that about 1-deg angle of attack must be added to the aircraft to maintain altitude (plus probably some added fuel consumption). Obviously, the added angle of attack is required to compensate for the downwash in the wake brought about by the lift on the wing of the wake-generating aircraft. The observation by the pilots that about 1 deg must be added to the aircraft angle of attack confirms that significant downwash is still present in the wake. Because the effective angle of attack of the wing is probably around 10 deg for 1 g level flight, a 1 deg change indicates that vertical accelerations on the order of 1 deg/10 deg or 0.1 g might be experienced by quickly making a transverse penetration through the wake.

If, however, atmospheric conditions are such that the mutually induced instability does not occur and proceed to completion, the vortex wake could remain coherent and compact with a significantly larger downwash and rolling-moment hazard for a much longer time than usual. Recall that any piloted control inputs are predicted by McGowan[8] to aggravate the wake-induced accelerations. It is, therefore, strongly recommended that such a practice be stopped, that is, it is believed that at no time is it absolutely safe to enter

Fig. 21 Diagonal view from below of wake about 0.5 h old with lateral striations along with newly formed wake.

a vortex wake. The foregoing experiences do indicate that vortex wakes that have experienced decomposition through the mutually induced instability generally have very little if any coherent rolling moment and that the downwash has persisted.

Stage 4: Wake Reorganizes into Coherent Vortex Pair (5–15 Minutes)

Before this study began, it was thought that the eddies that had appeared on all sides of a wake would die down to produce a wake that appears as a long irregularly shaped and quiescent cloud of condensate. However, the wake shapes in Figs. 16–20 indicate that eddies observed during earlier stages are still somewhat active. Furthermore, the shape of the condensation trail does not appear to consist of a coherent vortex pair. It is not until the wake has aged about 20 min or more in time that the presence of a coherent vortex pair becomes apparent through lateral striations in the exhaust condensate (Fig. 21). The photographs chosen are presented in Figs. 3 and 21 because each had at least one relatively new wake (about 1 min old) in the same view to indicate the tremendous size of the striated wake at this stage, that is, 30–40 times the size of the relatively new wake, or about 100 times the span of the wake-generating aircraft. Also note that the new wake disappears as it passes over the more mature wake as if the older wake disrupts the condensation process. In addition, the offset in the older wake suggests that the new wake may be influencing its location and shape. The large wake probably does not pose any real hazard to encountering aircraft, but it is of interest from a basic fluid mechanics point of view. If the lateral striations that emanate from the centerline condensate cloud are brought about by a vortex pair, the arrangement of vortices and condensate would appear in end view something like the flowfield shown in Fig. 22. The indicated flowfield does not explain how the vortex became so large, nor how the vortex separated from the condensate-containing air and then formed on top of the visible wake. Buoyancy of the various parts of the flowfield and of the atmosphere probably play a significant role in the development of the final flowfield structure.

Stage 5: Condensation Striations Grow in Length (15 Minutes–1 Hour)

After the feather-shaped striations were observed, efforts were made to find wake shapes that illustrate the transition process from

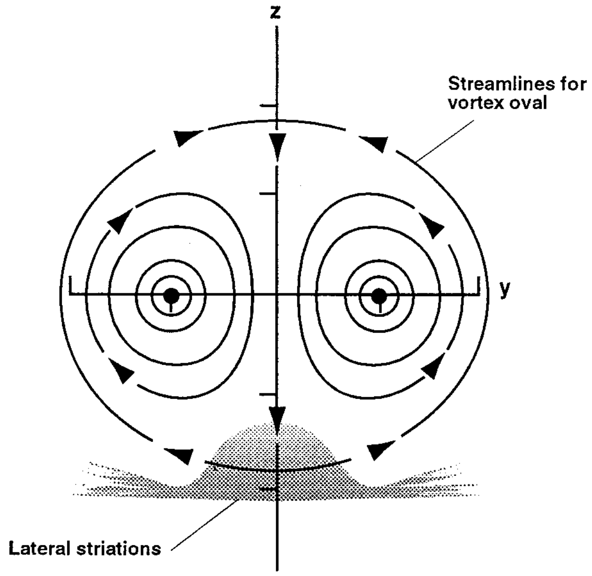
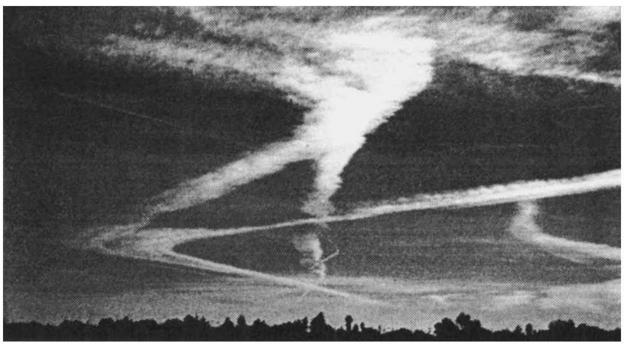


Fig. 22 Suggested flowfield interpretation of condensation wakes with lateral striations.



a)

b)

Fig. 23 Two examples of condensation wakes about 30 min old in the early stages of forming lateral striations like those in Fig. 21.

the earlier stage to the striated stage. In Fig. 23a, several of the wakes appear to be on their way to generating striations, but well-defined striations did not appear before a general cloud cover obscured all of the trails. The wake in Fig. 23b is included because horizontal spreading of the wake, with very little vertical growth, suggests that striations may be in the process of forming. The condensation wake being shed by aircraft passing near the older wake is again a good indication of the size of the vortex pairs that are likely just above the striations. The striations of condensate always seem to be connected to the condensate cloud at the center of the wake. Sometimes condensate accumulates at the outer ends of the striations as if air-containing condensate were being swept to the extremities of the striations (Fig. 3). At no time do the striations appear as if they have their origin at the outboard edges of the flowfield of the vortex pair. One plausible explanation for the formation of the spanwise striations (rather than a continuous sheet of condensate) is that the bunching of fluid by the vortex loops formed during the Crow instability[45] brings about an irregular shape to the condensate cloud that is swept laterally to form the striations. Such a process must depend strongly on atmospheric stability to hold all of the components of the condensate pattern at about the same altitude.

If the flowfield model suggested in Fig. 22 is correct, the condensate striations grow in length in response to the presence and nearness of a vortex pair centered just over the striations. The downward momentum from the lift on the generating wing could be responsible for the reformation of the wake into a pair of vortices, but why the vortices grow to such a large size even though apparently quite coherent is not certain. The flowfield shown in Fig. 22 does not explain the mechanism for the growth of the vortex pair to such a large size nor for the origin and shape of the striations. As this stage of vortex change proceeds, condensate sometimes accumulates at the spanwise extremities of the striations (Fig. 3). It is suspected that buoyancy of the fluid and atmospheric stratification are necessary for the definition and structure of the flowfield.

Processes other than a lift-generated wake of an aircraft were considered as the source of these unusual formations of condensation clouds. A close association of the locations of events with the passage of lift-generated aircraft suggests strongly that the feathertype formations are a consequence of aging vortex wakes. Unfortunately, background haze and cirrus clouds in the region of observations made some of the photographs unacceptable for publication. Both ambient turbulence and wind shear must be very near zero for such fine detail to be produced over such long periods of time.

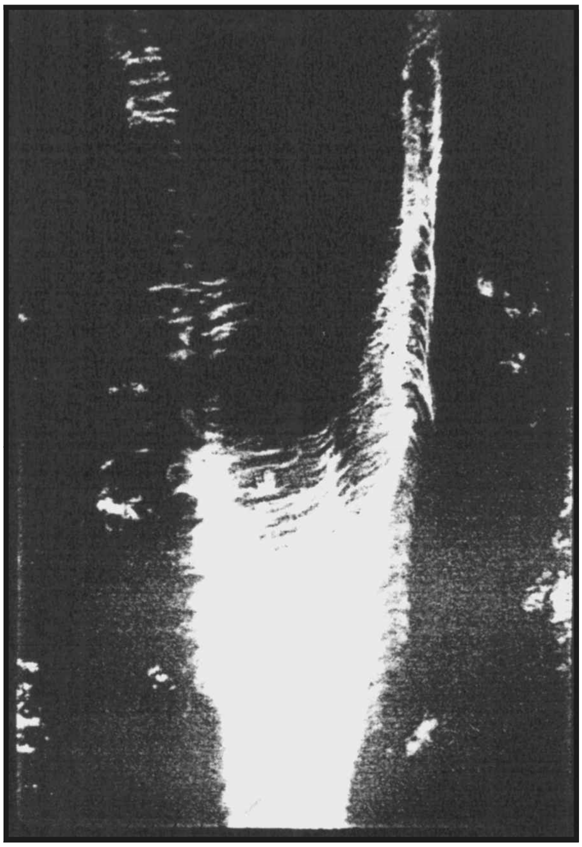
Theoretical analyses of the dynamics of vortex wakes as they descend into a stratified fluid have been studied for some time.[50–57] Any analysis of the long-term behavior of vortex wakes should follow the development of the wake for lengths of time comparable with the observations reported here. It should then be determined whether or not inclusion of the mutually induced and other instabilities in the computations cause differences in the final result. When the aerodynamic mechanisms responsible for the huge growth of the vortex wake are explained, methods for achieving more rapid growth in vortex wakes in the terminal environment may be suggested.

If the foregoing estimate of the structure of vortex wakes at such an advanced age is correct, the swirling velocities have been reduced by vortex size growth to below 0.01 times their value in the outer parts of the original vortex oval. If the swirl velocities in the intense core regions of the vortices are assumed to have been dissipated by the mutually induced and other instabilities during the early history of the wake, velocities much in excess of 1 ft/s are not expected. Therefore, it is probably safe to assume that at this stage the wake does not pose either an in-trail nor an across-trail hazard.

Stage 6: Condensation Trail Loses Identity (After 1 Hour)

Observations of condensation wakes indicate that they disappear for a variety of reasons. When conditions for the formation of condensation wakes are marginal, the wakes often do not appear at all or only for short periods of time after formation. When conditions are very good for the formation of condensation wakes, they persist until they mix with other wakes, cirrus clouds, or clouds or haze forming in the area. As condensation wakes persist for long periods of time, the special structures identified in the figures presented here seem to occur. Because these observations all occurred during low- or no-wind conditions, atmospheric or other types of wake breakup was not observed. However, as the size of the coherent vortex driving the striations becomes larger, the magnitude of the velocity components in the flowfield must also diminish linearly with size. Therefore, at some time as the vortices grow in size, the wake velocities must eventually decrease to a level where ambient disturbances will overwhelm the organization of the wake inside of the very large vortex oval. This either was not observed or was not recognized when it did occur.

One unexpected event in the later stages of wake demise was a division of the vortex axis from one into two parts (Fig. 24a). It seems likely that the axes in the vortex pair diverge because the wake descended down to a level in the atmosphere that acts like a ground plane. As the vortex pair approaches the atmospheric, or virtual, ground plane, the self-induced velocity field of the vortices



a)

b)

Fig. 24 Photographs of condensation wake with lateral striation after a segment of the trail has divided to produce two trails: a) after wake has divided to form two approximately similar branches and b) about 10 min later, curvature of one branch has increased so that it makes two right angle turns.

(and their images) causes them to move laterally away from each other, as shown the upper part of Fig. 24a. A photograph taken about 10 min later (Fig. 24b) indicates that not only do the wakes separate into two separate parts, but one branch takes on a sharp bend that increases with time. Also apparent in Fig. 24 is that each vortex must also have induced a secondary vortex in the ambient fluid, so that a stagnation line forms along the two branches of the wake.

Modification of Aircraft to Alleviate Loads

The prospect of modifying the wake-generating aircraft so that it does not shed a hazardous wake has been studied since around 1970. Some of these theoretical studies and experimental results indicate that the dissipation or dispersion of vortex wakes can be accomplished by special wing designs, so that the rolling-moment hazard they pose can be significantly reduced for approach configurations of transport aircraft. The techniques studied employ various devices and aerodynamic processes to render the intense centers of the vortices incoherent and, therefore, less hazardous.[6]

A comparable demonstration of significant alleviation of the hazard posed by the downwash of aircraft wakes at cruise or in the airport vicinity has not been demonstrated nor have methods been suggested for more rapid spreading of the downwash. The spreading of the downwash is believed to be a much more difficult problem to solve than the one associated with the rolling-moment hazard. The reason for the difference is that the downward momentum of air persists in the wake even if the centers of the vortices are made incoherent because most of the downwash is concentrated in a region on the order of the span of the wing. The most obvious way to reduce the downwash velocities is to go to larger wing spans. However, even a doubling of the wing span for a given weight of aircraft would only reduce the vertical loads by a factor of 2, whereas it is estimated that at least a factor of 10 is needed. An aerodynamic solution to the downwash and across-trail penetration problem requires the invention of some sort of way to tailor the span loading on the wake-generating wing so that the downwash distribution spreads much more rapidly than it does with conventional wing designs.

Wake-Avoidance Procedure

At this time, the vortex wakes of aircraft change slowly so that up to about 20 min for example, 150–200 miles behind the generating aircraft, is required before wakes are safe to penetrate in any direction and their presence ignored. Based on the limited and remote observations presented here, if aircraft avoid a 200-mile-long tube of the atmosphere that begins at about one span in diameter and increases linearly with distance to about 20 spans in diameter, hazardous wake encounters should not occur. The cross section of the hazardous regions to be used in the computations that monitor locations of hazardous regions needs to be large enough to include the entire wake and a sufficient safety factor that allows for errors in predicting the location of each cross section and to include other unknowns or unanticipated factors.

Improved precision of the flight paths used by aircraft to approach and depart airports, and while in cruise, is available on many aircraft by use of the global positioning system (GPS). It is anticipated that the accuracy of location will be improved even further when the present second frequency and a third new frequency become available to the public. Even with only the single frequency in use at this time, sufficient accuracy is available for accurate estimates for the locations of lift-generated wakes and their associated hazard as a function of time. When such a procedure is studied,[58] it is found that the first step is to estimate the size of the hazardous region around the wake of the wake-generating aircraft. Information contained in this paper provides a starting point for a determination that includes both time and distance behind the generating aircraft. To determine wake motion after generation, the aircraft must also monitor the wind velocities along its flight path. By use of the initial location of the vortex wake and the measured winds along the flight path, the location of the hazardous part of the wake can be updated periodically, for example, every 1–5 s. This information is then disseminated in GPS coordinates to other aircraft so that each can avoid the hazardous regions thus identified and recorded in the monitoring system. The process involves a lot of time-dependent bookkeeping that can be easily handled by existing computers. Before implementation, the dimensions of the tube that encloses the hazardous region in an actual system should be thoroughly tested to be certain that the foregoing estimates are safe and that all known factors have been considered and evaluated.

Conclusions

Based on observations of lift-generated wakes made visible by condensation of exhaust gases, changes in wakes due to aging are followed until the wakes appear to become very weak or to blend with the ambient atmosphere. Equations based on the transfer of momentum from the wake to the penetrating aircraft during a wake encounter appear to yield results in good agreement with more complete analyses. The relationships presented are probably accurate enough for the encounter problem because they are in good agreement with more complete analyses, and uncertainties in the size of the wake appear to be large.

On the basis of the study, the hazard posed by the downwash in the wake of aircraft appears to fade away slowly with time as the wake spreads laterally and vertically. Estimates of the loads that may occur during a wake encounter indicate that the intense rotary velocities associated with the wake-vortex hazard become benign within several minutes after the passage of the wake-generating aircraft, that is, about 20 miles. Based on the observed cross-sectional sizes of wakes as they age, and on estimates of vertical loads based on momentum transfer, it is expected that a hazardous level persists for wake crossings at cruise altitudes for times up to about 20 min (up to 200 miles) after being generated.

The data obtained during this study indicate that at no time should an intentional penetration of a vortex wake be made because wake structures, and the decay with time, can vary over wide limits, so that it is not possible to predict when it is safe to penetrate a wake in any direction. Furthermore, because the present study did not carry out direct measurements of vortex wakes at cruise altitudes, it is recommended that wakes at cruise altitudes be probed for their velocity distributions as a function of time to obtain a more accurate assessment of the hazards they pose, in both space and time. The observations of condensation trails of aircraft at cruise altitude identified several wake-vortex instabilities and growth mechanisms that are not well understood. Analysis of the aerodynamic processes that may be present and more high-quality and frequent photographs of condensation wakes are needed to help diagnose the reasons behind some of the striated formations in aircraft condensation wakes.

Because condensation wakes are not always visible so that they can be avoided by aircraft, it is recommended that an avoidance procedure be developed to keep track of the location of each wake and their size in both altitude and geographical location. At this time, utilization of the GPS makes such an avoidance system feasible.

References

[1]Olsen, J. H., Goldburg, A., and Rogers, M., eds., *Aircraft Wake Turbulence and its Detection*, Plenum, New York, 1971.

[2]Gessow, A., ed., *NASA Symposium on Wake Vortex Minimization*, SP-409, NASA, Feb. 1976.

[3]Hallock, J. N., ed., *Proceedings of the Aircraft Wake Vortices Conference*, Rept. FAA-RD-77-68, U.S. Dept. of Transportation, June 1977.

[4]Wood, W. D., ed., *FAA/NASA Proceedings Workshop on Wake Vortex Alleviation and Avoidance*, Rept. FAA-RD-79-105, U.S. Dept. of Transportation, Oct. 1978.

[5]Hallock, J. N., ed., *Proceedings of the Aircraft Wake Vortices Conference*, Vols. 1 and 2, DOT/FAA/SD-92/1.1, DOT-VNTSC-FAA-92-7.1, U.S. Dept. of Transportation, Federal Aviation Administration, Washington, DC, 1991.

[6]Rossow, V. J., "Lift-Generated Vortex Wakes of Subsonic Transport Aircraft," *Progress in Aerospace Sciences*, Vol. 35, No. 6, 1999, pp. 507–660.

[7]Hallock, J. N., "Aircraft Wake Vortices: An Annotated Bibliography (1923–1990)," Rept. DOT-FAA-RD-90-30, DOT-VNTSC-FAA-90-7, John A. Volpe National Transportation Systems Center, U.S. Dept. of Transportation, Cambridge, MA, Jan. 1991.

[8]McGowan, W. A., "Calculated Normal Load Factors on Light Airplanes Traversing the Trailing Vortices of Heavy Transport Airplanes," NASA TN D-829, May 1961.

[9]Jones, R. T., "The Unsteady Lift of a Wing of Finite Aspect Ratio," NACA Rept. 681, 1940.

[10]Pierce, H. B., "Investigation of the Dynamic Response of Airplane Wings to Gusts," NACA TN 1320, June 1947.

[11]Kordes, E. E., and Houbolt, J. C., "Evaluation of Gust Response Characteristics of Some Existing Aircraft with Wing Bending Flexibility Included," NACA TN 2897, Feb. 1953.

[12]Rose, R., and Dee, F. W., "Aircraft Vortex Wakes and Their Effects on Aircraft," TN Aero. 2934, British Royal Aircraft Establishment, Farnborough, England, U.K., Dec. 1963.

[13]Bisgood, P. L., Maltby, R. L., and Dee, F. W., "Some Work at the Royal Aircraft Establishment on the Behaviour of Vortex Wakes," *Aircraft Wake Turbulence and its Detection*, edited by J. H. Olsen, A. Goldburg, and M. Rogers, Plenum, New York, 1971, pp. 171–206.

[14]Condit, P. M., and Tracy, P. W., "Results of The Boeing Company Wake Turbulence Test Program," *Aircraft Wake Turbulence and its Detection*, edited by J. H. Olsen, A. Goldburg, and M. Rogers, Plenum, New York, 1971, pp. 473–508.

[15]Houbolt, J. C., "Aircraft Response to Turbulence Including Wakes," *Aircraft Wake Turbulence and its Detection*, edited by J. H. Olsen, A. Goldburg, and M. Rogers, Plenum, New York, 1971, pp. 509–522.

[16]Jones, W. P., and Rao, B. M., "Airloads and Moments on an Aircraft Flying Over a Pair of Inclined Trailing Vortices," *Aircraft Wake Turbulence and its Detection*, edited by J. H. Olsen, A. Goldburg, and M. Rogers, Plenum, New York, 1971, pp. 523–545.

[17]Ramirez, A., Jr., Rao, B. M., and Cronk, A. E., "Longitudinal Response of an Aircraft Due to a Trailing Vortex," *Journal of Aircraft*, Vol. 10, No. 10, 1973, pp. 638, 639.

[18]Nelson, R. C., "The Dynamic Response of Aircraft Encountering Aircraft Wake Turbulence," Air Force Flight Dynamics Lab., TR AFFDL-TR-74-29, Wright–Patterson AFB, OH, June 1974.

[19]Nelson, R. C., "Dynamic Behavior of an Aircraft Encountering Aircraft Wake Turbulence," *Journal of Aircraft*, Vol. 13, No. 9, 1976, pp. 704–708.

[20]McWilliams, I. G., "Hazard Extent About Aircraft Trailing Wake Vortices—Analytic Approach," *Proceedings of the Aircraft Wake Vortices Conference*, edited by J. N. Hallock, Rept. FAA-RD-77-68, Transportation Systems Center, U.S. Dept. of Transportation, Cambridge, MA, 1977, pp. 23–30.

[21]Britton, J. W., "Some Remarks on En-Route Vortex Encounters," *Proceedings of the Aircraft Wake Vortices Conference*, edited by J. N. Hallock, Rept. FAA-RD-77-68, Transportation Systems Center, U.S. Dept. of Transportation, Cambridge, MA, 1977, pp. 243–246.

[22]Pinsker, W. J. G., "The Hazard of Vortex Wake Encounters in the Cruise," Royal Aircraft Establishment, TR 79063, Farnborough, England, U.K., June 1979.

[23]News Breaks, "At Least Six People Were Injured," *Aviation Week and Space Technology*, Vol. 150, No. 21, 1998, p. 16.

[24]Cha, A. E., "Turbulence Buffets Airliner Over Pacific; Passenger Dies—Trouble Comes Without Warning," *San Jose Mercury News*, Vol. 147, No. 193, 1997, p. 1.

[25]Minnick, W., "Operation Free Flight—An Operational Evaluation of RNAV Direct Route Flight Plan Filing in Today's National Airspace System," Federal Aviation Administration, Rept. FAA-AT-81-1, Washington, DC, July 1981.

[26]"Final Report of RTCA Task Force 3, Free Flight Implementation," RTCA, Inc., Washington, DC, Oct. 1995.

[27]Pilie, R. J., and Jiusto, J. E., "A Laboratory Study of Contrails," *Journal of Meteorology*, Vol. 15, No. 2, 1958, pp. 149–154.

[28]Scorer, R. S., and Davenport, L. J., "Contrails and Aircraft Downwash," *Journal of Fluid Mechanics*, Vol. 43, 1970, pp. 451–464.

[29]Sassen, K., "Contrail–Cirrus and Their Potential for Regional Climate Change," *Bulletin of the American Meteorological Society*, Vol. 78, No. 9, 1997, pp. 1885–1903.

[30]Sassen, K., and Dodd, G. C., "Haze Particle Nucleation Simulations in Cirrus Clouds and Applications for Numerical and Lidar Studies," *Journal of Atmospheric Sciences*, Vol. 46, No. 19, 1989, pp. 3005–3014.

[31]Mims, F. M., and Travis, D. J., "Aircraft Contrails Reduce Solar Irradiance," *EOS, Transactions*, American Geophysical Union, Vol. 78, No. 41, 1997, p. 448.

[32]Ciffone, D. L., and Orloff, K. L., "Far-Field Wake-Vortex Characteristics of Wings," *Journal of Aircraft*, Vol. 12, No. 5, 1975, pp. 464–470.

[33]Iversen, J. D., "Correlation of Turbulent Trailing Vortex Decay Data," *Journal of Aircraft*, Vol. 13, No. 5, 1976, pp. 338–342.

[34]Corsiglia, V. R., Rossow, V. J., and Ciffone, D. L., "Experimental Study of the Effect of Span Loading on Aircraft Wakes," *Journal of Aircraft*, Vol. 13, No. 12, 1976, pp. 968–973.

[35]Corsiglia, V. R., and Dunham, R. E., "Aircraft Wake-Vortex Minimization by Use of Flaps," *NASA Symposium on Wake Vortex Minimization*, SP-409, NASA, 1976, pp. 305–338.

[36]Rossow, V. J., Corsiglia, V. R., Schwind, R. G., Frick, J. K. D., and

Lemmer, O. J., "Velocity and Rolling-Moment Measurements in the Wake of a Swept-Wing Model in the 40- by 80-Foot Wind Tunnel," NASA TM X-62,414, April 1975.

[37]Rossow, V. J., Sacco, J. N., Askins, P. A., Bisbee, L. S., and Smith, S. M., "Measurements in 80 by 120 Foot Wind Tunnel of Hazard Posed by Lift-Generated Wakes," *Journal of Aircraft*, Vol. 32, No. 2, 1995, pp. 278–284.

[38]Rossow, V. J., Fong, R. K., Wright, M. S., and Bisbee, L. S., "Vortex Wakes of Two Transports Measured in 80 by 120 Foot Wind Tunnel," *Journal of Aircraft*, Vol. 33, No. 2, 1996, pp. 399–406.

[39]Rossow, V. J., "Estimate of Loads During Wing–Vortex Interactions by Munk's Transverse-Flow Method," *Journal of Aircraft*, Vol. 27, No. 1, 1990, pp. 66–74.

[40]Jones, R. T., and Cohen, D., "Aerodynamics of Wings at High Speeds," Vol. 7, *High Speed Aerodynamics and Jet Propulsion—Aerodynamic Components of Aircraft at High Speeds*, edited by A. F. Donovan and H. R. Lawrence, Princeton Univ. Press, Princeton, NJ, 1957.

[41]Rossow, V. J., "Validation of Vortex-Lattice Method for Loads on Wings in Lift-Generated Wakes," *Journal of Aircraft*, Vol. 32, No. 6, 1995, pp. 1254–1262.

[42]Glauert, H., *The Elements of Aerofoil and Airscrew Theory*, Cambridge Univ. Press, Cambridge, England, U.K., 1948.

[43]Crow, S. C., "Stability Theory for a Pair of Trailing Vortices," *AIAA Journal*, Vol. 8, No. 12, 1970, pp. 2172–2179.

[44]Lamb, Sir H., *Hydrodynamics*, 6th ed., Dover, New York, 1945, p. 592.

[45]Crow, S. C., and Bate, E. R., Jr., "Lifespan of Trailing Vortices in a Turbulent Atmosphere," *Journal of Aircraft*, Vol. 13, No. 7, 1976, pp. 476–482.

[46]Laporte, F., Darracq, D., and Corjon, A., "On the Vortex-Turbulence Interaction: DNS of Elliptic Instability of a Vortex Pair," AIAA Paper 99-3417, June 1999.

[47]Crouch, J. D., "Instability and Transient Growth for Two Trailing-Vortex Pairs," *Journal of Fluid Mechanics*, Vol. 350, Nov. 1997, pp. 311–330.

[48]Leweke, T., and Williamson, C. H. K., "Cooperative Elliptic Instability of a Vortex Pair," *Journal of Fluid Mechanics*, Vol. 360, 1998, pp. 85–119.

[49]Greene, G. C., "An Approximate Model of Vortex Decay in the Atmosphere," *Journal of Aircraft*, Vol. 23, No. 7, 1986, pp. 566–573.

[50]Saffman, P. G., "The Motion of a Vortex Pair in a Stratified Atmosphere," *Studies in Applied Mathematics*, Vol. 51, June 1972, pp. 107–119.

[51]Hill, F. M., "A Numerical Study of the Descent of a Vortex Pair in a Stably Stratified Atmosphere," *Journal of Fluid Mechanics*, Vol. 71, Pt. 1, 1975, pp. 1–13.

[52]Hecht, A. M., Bilanin, A. J., and Hirsch, J. E., "Turbulent Trailing Vortices in Stratified Fluids," *AIAA Journal*, Vol. 19, No. 6, 1981, pp. 691–698.

[53]Sarpkaya, T., "Trailing Vortices in Homogeneous and Density-Stratified Media," *Journal of Fluid Mechanics*, Vol. 136, Nov. 1983, pp. 85–109.

[54]Neuhart, D. H., Greene, G. C., Satran, D. R., and Holbrook, G. T., "Density Stratification Effects on Wake Vortex Decay," *Journal of Aircraft*, Vol. 23, No. 11, 1986, pp. 820–824.

[55]Spalart, P. R., "On the Motion of Laminar Wing Wakes in a Stratified Fluid," *Journal of Fluid Mechanics*, Vol. 327, Nov. 1996, pp. 139–160.

[56]Robins, R. E., and Delisi, D. P., "Numerical Simulation of Three-Dimensional Trailing Vortex Evolution in Stratified Fluid," *AIAA Journal*, Vol. 36, No. 6, 1998, pp. 981–985.

[57]Holzaepfel, F., Gerz, T., and Baumann, R., "The Turbulent Decay of Wake Vortices in the Stably Stratified Atmosphere," AIAA Paper 2000-0754, Jan. 2000.

[58]Rossow, V. J., "Wake-Vortex Separation Distances when Flight-Path Corridors are Constrained," *Journal of Aircraft*, Vol. 33, No. 3, 1996, pp. 539–546.